Math 112 - Test 1
February 13, 2019

Name key

Score _____

Show all work. Supply explanations when necessary. Partial credit will be awarded for correct work.

1. (6 points) Write each set in roster notation.

   (a) A is the set of counting numbers less than 6.

   $\{1, 2, 3, 4, 5\}$

   (b) Q is the set of letters of the word *sleeplessness*.

   $\{s, l, e, p, n\}$

   (c) $D = \{\dot{s} \mid s \in \mathbb{N} \text{ and } 6 < s < 10\}$

   $\{7, 8, 9\}$

2. (2 points) The set of heart failure patients who deserve heart transplants is not well defined. Give one reason for why not.

   IT IS NOT CLEAR WHAT THE WORD "DESERVE" MEANS IN THIS CONTEXT. IT SEEMS LIKE THAT IS SUBJECTIVE RATHER THAN OBJECTIVE.

3. (3 points) The following set is described in set-builder notation. Translate this description word for word into a complete sentence.

   $$B = \{x \mid x \in \mathbb{N} \text{ and } x < 10\}$$

   B IS THE SET OF ALL X SUCH THAT X IS AN ELEMENT OF THE NATURAL NUMBERS AND X IS LESS THAN 10.

4. (4 points) Give a verbal description of each set.

(a) $\{10, 12, 14, 16, \ldots, 48\}$

THE SET OF EVEN COUNTING NUMBERS FROM 10 TO 48.

(b) $\{a, u, g, s, t\}$

THE SET OF LETTERS OF THE WORD August.

5. (2 points) Write the set $M = \{1, 2, 3, 4, \ldots, 100\}$ in set-builder notation.

$M = \{x \mid x \in \mathbb{N} \text{ AND } x \leq 100\}$

6. (5 points) Decide whether each statement is true or false.

(a) T $79 \in \{1, 3, 5, 7, 9, 11, \ldots\}$ 79 IS AN ODD #

(b) F $-5 \in \mathbb{N}$ NEGATIVES ARE NOT COUNTING #'s

(c) T $\{r \mid r \in \mathbb{N} \text{ and } r + 1 = 0\}$ is the empty set. $\emptyset$

$r+1=0 \Rightarrow r=-1$

(d) F $\{k \mid k \in \mathbb{N} \text{ and } k - 1 = 0\}$ is the empty set. $\{1\}$

$k-1=0 \Rightarrow k=1$

(e) F Homewood $\in \{x \mid x \text{ is one of the United States}\}$

↑ CITY, NOT A STATE

7. (5 points) Determine the cardinal number for each set.

(a) 5 $A = \{2, 4, 6, 8, 10\}$

(b) 6 B = the set of letters of the word *didgeridoo* $\{d, i, g, e, r, o\}$

(c) 1 $C = \{\emptyset\}$

(d) 1 $B = \{\text{four}\}$

(e) ∞ $E = \mathbb{N}$

8. (2 points) Give an example of a set that is equivalent to, but not equal to, $\{1, 2, 3, 4\}$.

Any 4-element set that is not

$\{a, b, c, d\}$

9. (4 points) List all subsets of the set $\{1, 2, 3\}$.

$\emptyset$, $\{1\}$, $\{2\}$, $\{3\}$,

$\{1,2\}$, $\{1,3\}$, $\{2,3\}$, $\{1,2,3\}$

There are 8 subsets.

10. (4 points) Decide whether each statement is true or false.

(a) T $\{3\} \cong \{\emptyset\}$

(b) T All equal sets are equivalent.

(c) F All equivalent sets are equal.

(d) T $\{1, 2, 3, 4, 5\} \cong \{10, 20, 30, 40, 50\}$

11. (1 point) Let $A = \{d, o, g\}$. Which one of these sets is NOT a proper subset of A?

(a) $\{d\}$
(b) $\{d, o, g\}$
(c) $\{d, g\}$
(d) $\emptyset$

12. (1 point) Let $C = \{1, 2, 3, 4, 5, 6, 7\}$. How many subsets does C have?

(a) 7
(b) 100
(c) 128
(d) 150

$2^7 = 128$

13. (1 point) Which one of these sets is equivalent to $\{a, b, c\}$?

(a) $\{abc\}$
(b) $\{123\}$
(c) $\{1, 2, 3\}$
(d) $\emptyset$

MUST HAVE 3 ELEMENTS

14. (1 point) Suppose A and B are NOT empty sets. Which one of the following sets IS empty?

(a) $\{\emptyset\}$
(b) $A \cup B$
(c) $B \cap B'$
(d) $A \cup B \cup \emptyset$

NOTHING IN BOTH B AND B'

15. (1 point) Let $A = \{1, 2\}$ and $B = \{a, b, c\}$. Which one of these is an element of $A \times B$?

(a) $\{1, 2, a, b, c\}$
(b) $\{1, b\}$
(c) $(1, 1)$
(d) $(2, c)$

16. (16 points) Let U be the set of letters of the English alphabet and think about the subsets $A = \{a, b, c, d, e\}$ and $B = \{a, e, i, o, u\}$. Determine each of the following.

(a) $n(B)$

$= 5$

(b) $A' = \{f, g, h, i, \ldots, z\}$

(c) $A \cup B = \{a, b, c, d, e, a, i, o, u\}$

(d) $A \cap B = \{a, e\}$

(e) $A \cap B' = \{b, c, d\}$

(f) $A - B = \{b, c, d\}$ (Same as (e))

(g) $\emptyset \cup B = B = \{a, e, i, o, u\}$

(h) $A \cap \emptyset = \phi$

17. (6 points) Suppose U is the set of dogs at the local animal shelter. Let G be the subset of gray dogs and let H be the subset of dogs weighing more than 50 lbs.

(a) How would you describe the elements of $G \cap H$?

SHELTER DOGS THAT ARE GRAY AND OVER 50 lbs

(b) How would you describe the elements of G'?

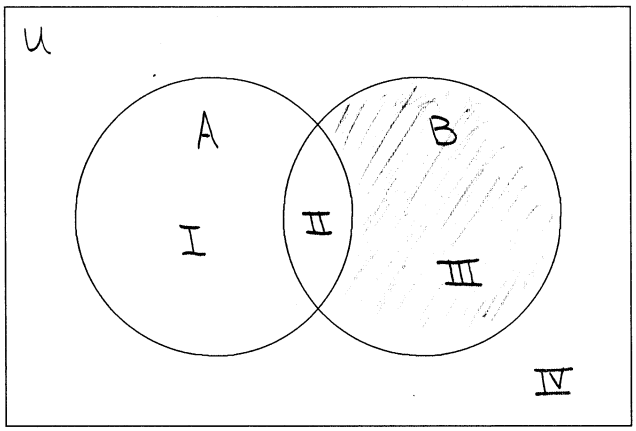
SHELTER DOGS THAT ARE NOT GRAY.

18. (6 points) In the two-set Venn diagram shown below, label the sets A and B. Then label the four distinct (disjoint) regions with Roman numerals. Identify and shade the regions that make up $B - A$.

$B = \{II, III\}$

$A = \{I, II\}$

$B - A = \{III\}$



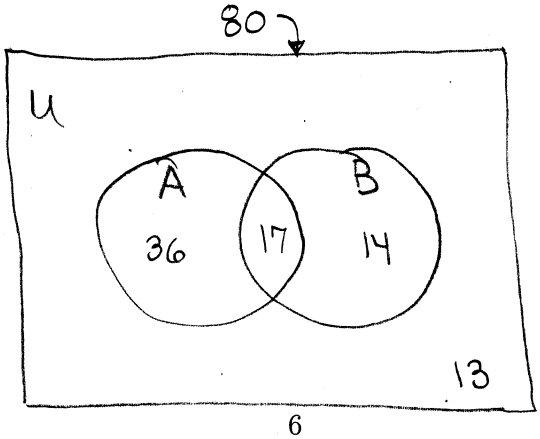
19. (8 points) Suppose $n(A) = 53$, $n(B) = 31$, $n(A \cap B) = 17$, and $n(U) = 80$. Use a Venn diagram to sort the data. Then determine $n(A \cup B)$.

II: 17

I: 53 - 17 = 36

III: 31 - 17 = 14

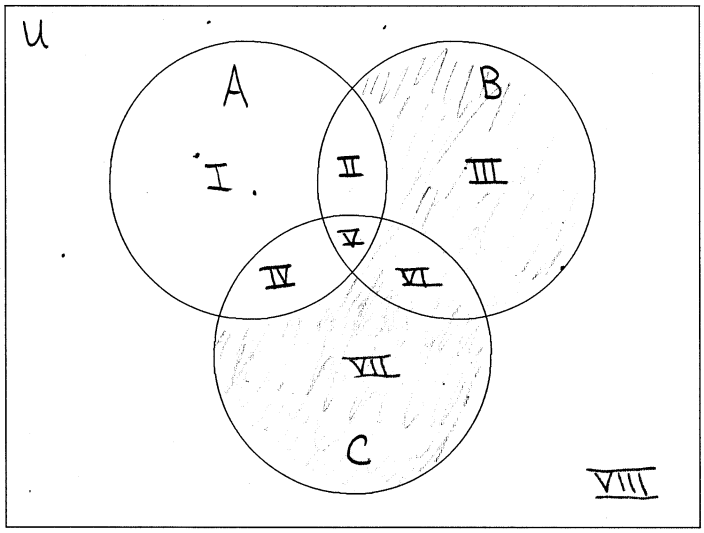
IV: 80 - (36 + 17 + 14)
= 80 - 67 = 13



$n(A \cup B) = 36 + 17 + 14$

$= \boxed{67}$

20. (8 points) In the three-set Venn diagram shown below, label the sets A, B, and C. Then label the distinct (disjoint) regions of the diagram with Roman numerals. Identify and shade the regions that make up $A' \cap (B \cup C)$.



$A' = \{ \text{III}, \text{VI}, \text{VII}, \text{VIII} \}$ (OUTSIDE A)

$B \cup C = \{ \text{II}, \text{III}, \text{V}, \text{VI}, \text{VII} \}$ (B OR C OR BOTH)

$A' \cap (B \cup C) = \{ \text{III}, \text{VI}, \text{VII} \}$

21. (6 points) Rewrite each of the following statements using mathematical symbols.

(a) A is a subset of B.

$$A \subseteq B$$

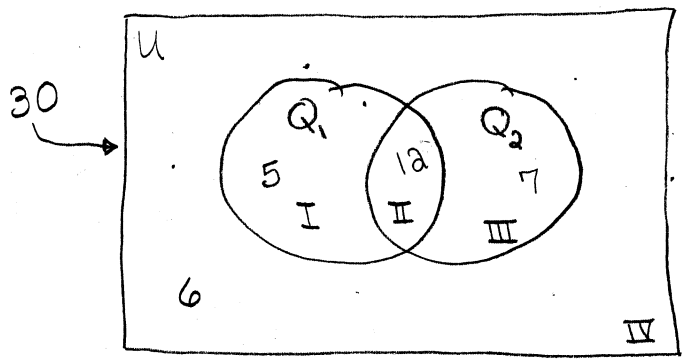
(b) The cardinal number of D is 10.

$$n(D) = 10$$

(c) a is an element of the complement of the set Q.

$$a \in Q'$$

22. (8 points) Out of 30 students taking an exam, 17 answered the first bonus question (Q1), 19 answered the second bonus question (Q2), and 6 did not attempt either of the two bonus questions. Use a Venn diagram to sort the data. Then determine how many students answered both bonus questions?



I + II + III --- 30 - 6 = 24 ELEMENTS

I + II --- 17 ELEMENTS

II + III --- 19 ELEMENTS

THESE GIVE

24 - 19 = 5 ELEMENTS IN I

WHICH MEANS 17 - 5 = 12 ELEMENTS IN II

AND 19 - 12 = 7 ELEMENTS IN III.

12 ANSWERED BOTH