Math 112 - Test 1

February 13, 2019

Name _____

Score _____

Show all work. Supply explanations when necessary. Partial credit will be awarded for correct work.

1. (6 points) Write each set in roster notation.

   (a) A is the set of counting numbers less than 6.

   (b) Q is the set of letters of the word *sleeplessness*.

   (c) $D = \{s \mid s \in \mathbb{N} \text{ and } 6 < s < 10\}$

2. (2 points) The set of heart failure patients who deserve heart transplants is not well defined. Give one reason for why not.

3. (3 points) The following set is described in set-builder notation. Translate this description word for word into a complete sentence.

$$B = \{x \mid x \in \mathbb{N} \text{ and } x < 10\}$$

4. (4 points) Give a verbal description of each set.

   (a) $\{10, 12, 14, 16, \ldots, 48\}$

   (b) $\{a, u, g, s, t\}$

5. (2 points) Write the set $M = \{1, 2, 3, 4, \ldots, 100\}$ in set-builder notation.

6. (5 points) Decide whether each statement is true or false.

   (a) _____ $79 \in \{1, 3, 5, 7, 9, 11, \ldots\}$

   (b) _____ $-5 \in \mathbb{N}$

   (c) _____ $\{r \mid r \in \mathbb{N} \text{ and } r + 1 = 0\}$ is the empty set.

   (d) _____ $\{k \mid k \in \mathbb{N} \text{ and } k - 1 = 0\}$ is the empty set.

   (e) _____ Homewood $\in \{x \mid x \text{ is one of the United States}\}$

7. (5 points) Determine the cardinal number for each set.

   (a) _____ $A = \{2, 4, 6, 8, 10\}$

   (b) _____ $B =$ the set of letters of the word *didgeridoo*

   (c) _____ $C = \{\emptyset\}$

   (d) _____ $B = \{\text{four}\}$

   (e) _____ $E = \mathbb{N}$

8. (2 points) Give an example of a set that is equivalent to, but not equal to, $\{1, 2, 3, 4\}$.

9. (4 points) List all subsets of the set $\{1, 2, 3\}$.

10. (4 points) Decide whether each statement is true or false.

    (a) _____ $\{3\} \cong \{\emptyset\}$

    (b) _____ All equal sets are equivalent.

    (c) _____ All equivalent sets are equal.

    (d) _____ $\{1, 2, 3, 4, 5\} \cong \{10, 20, 30, 40, 50\}$

11. (1 point) Let $A = \{d, o, g\}$. Which one of these sets is NOT a proper subset of A?

    (a) $\{d\}$
    (b) $\{d, o, g\}$
    (c) $\{d, g\}$
    (d) $\emptyset$

12. (1 point) Let $C = \{1, 2, 3, 4, 5, 6, 7\}$. How many subsets does C have?

    (a) 7
    (b) 100
    (c) 128
    (d) 150

13. (1 point) Which one of these sets is equivalent to $\{a, b, c\}$?

    (a) $\{abc\}$
    (b) $\{123\}$
    (c) $\{1, 2, 3\}$
    (d) $\emptyset$

14. (1 point) Suppose A and B are NOT empty sets. Which one of the following sets IS empty?

    (a) $\{\emptyset\}$
    (b) $A \cup B$
    (c) $B \cap B'$
    (d) $A \cup B \cup \emptyset$

15. (1 point) Let $A = \{1, 2\}$ and $B = \{a, b, c\}$. Which one of these is an element of $A \times B$?

    (a) $\{1, 2, a, b, c\}$
    (b) $\{1, b\}$
    (c) $(1, 1)$
    (d) $(2, c)$

16. (16 points) Let U be the set of letters of the English alphabet and think about the subsets $A = \{a, b, c, d, e\}$ and $B = \{a, e, i, o, u\}$. Determine each of the following.

(a) $n(B)$

(b) A'

(c) $A \cup B$

(d) $A \cap B$

(e) $A \cap B'$

(f) $A - B$

(g) $\emptyset \cup B$

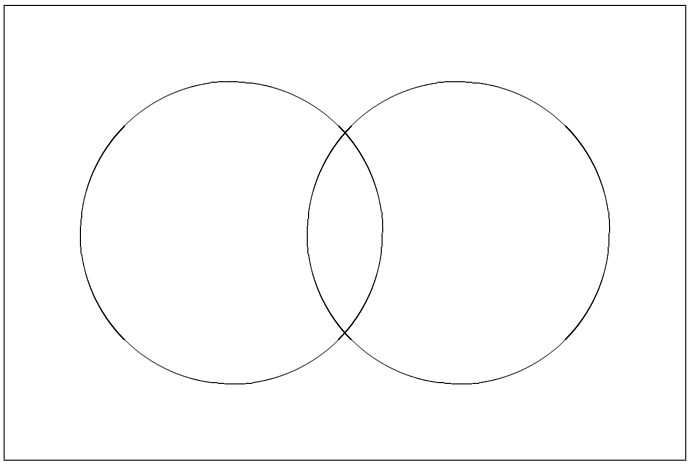
(h) $A \cap \emptyset$

17. (6 points) Suppose U is the set of dogs at the local animal shelter. Let G be the subset of gray dogs and let H be the subset of dogs weighing more than 50 lbs.

    (a) How would you describe the elements of $G \cap H$?

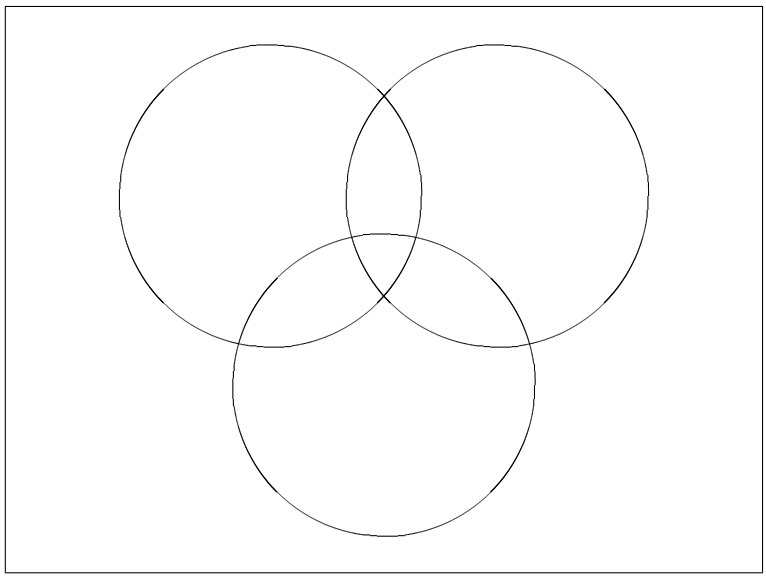
    (b) How would you describe the elements of G'?

18. (6 points) In the two-set Venn diagram shown below, label the sets A and B. Then label the four distinct (disjoint) regions with Roman numerals. Identify and shade the regions that make up $B - A$.



19. (8 points) Suppose $n(A) = 53$, $n(B) = 31$, $n(A \cap B) = 17$, and $n(U) = 80$. Use a Venn diagram to sort the data. Then determine $n(A \cup B)$.

20. (8 points) In the three-set Venn diagram shown below, label the sets A, B, and C. Then label the distinct (disjoint) regions of the diagram with Roman numerals. Identify and shade the regions that make up $A' \cap (B \cup C)$.



21. (6 points) Rewrite each of the following statements using mathematical symbols.

(a) A is a subset of B.

(b) The cardinal number of D is 10.

(c) a is an element of the complement of the set Q.

22. (8 points) Out of 30 students taking an exam, 17 answered the first bonus question (Q1), 19 answered the second bonus question (Q2), and 6 did not attempt either of the two bonus questions. Use a Venn diagram to sort the data. Then determine how many students answered both bonus questions?