

Math 112 - Final Exam A
May 14, 2019

Name key
Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (5 points) Circle the letter corresponding to each set that **IS NOT** well defined.

- (a) The set of human beings on Mars
- (b) The set of letters of the English alphabet
- ☒ (c) The set of all pretty flowers
- (d) The set of all natural numbers that are less than 0
- ☒ (e) The set of all smart students

2. (8 points) Let L be the set letters of the word *abracadabra*.

- (a) Write L in roster notation.

$$L = \{a, b, r, c, d\}$$

- (b) What is the cardinality of L ?

$$n(L) = 5$$

- (c) Give an example of a set that is equivalent to L , but not equal to L .

$$\{1, 2, 3, 4, 5\}$$

- (d) Give an example of a proper subset of L that has at least 2 elements.

$$\{a, b\}$$

3. (4 points) Rewrite each of the following statements using mathematical symbols.

- (a) The set A is a subset of the set of natural numbers.

$$A \subseteq \mathbb{N}$$

- (b) The empty set is an element of the set B .

$$\emptyset \in B$$

4. (12 points) Let $P = \{1, 3, 5\}$ and $Q = \{1, 2, 3, 5\}$, and think of P and Q as subsets of the universal set $U = \{1, 2, 3, 4, 5, 6\}$. Determine each of the following.

(a) $n(Q) = 4$

(b) $P' = \{2, 4, 6\}$.

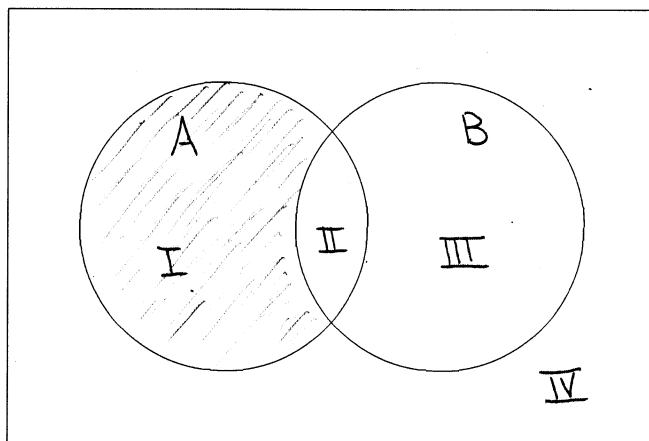
(c) $P \cup Q = \{1, 2, 3, 5\}$

(d) $P \cap Q = \{1, 3, 5\}$

(e) $P \div Q = \emptyset$

(f) $Q \cup \emptyset = \{1, 2, 3, 5\}$

5. (6 points) In the two-set Venn diagram shown below, label the sets A and B . Then label the four distinct (disjoint) regions with Roman numerals (or whatever system you prefer to use). Identify and shade the regions that make up $(A' \cup B)'$.



$$A' = \{\text{III}, \text{IV}\}$$

$$A' \cup B = \{\text{II}, \text{III}, \text{IV}\}$$

$$(A' \cup B)' = \{\text{I}\}$$

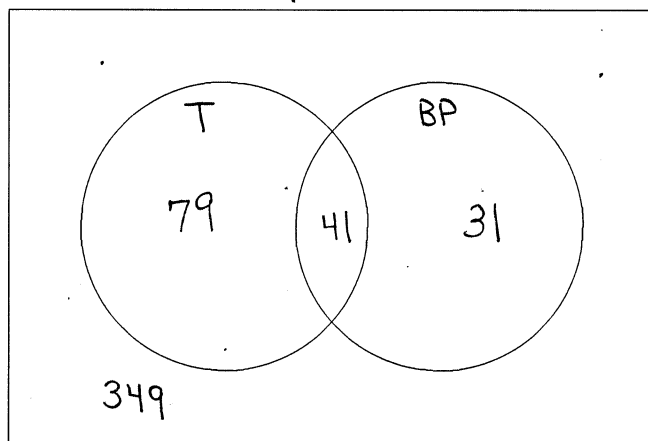
6. (10 points) In a survey, 500 randomly selected people were asked whether they have a tattoo and/or a body piercing. Here are the results:

- 79 have only a tattoo
- 31 have only a body piercing
- 151 have a tattoo or a body piercing (or both)

(a) Count and sort these results using a two-set Venn diagram.

$$151 - (79 + 31) \\ = 41$$

$$500 - 151 \\ = 349$$



(b) How many people have both a tattoo and a body piercing?

41

(c) How many people have neither a tattoo nor a body piercing?

349

7. (5 points) Which of these sentences are statements? Circle all that apply.

☒ (a) $3 \times 5 = 35$

☐ (b) Just do it!

☒ (c) Marie Curie was awarded the 1903 Nobel Prize in Physics.

☐ (d) Do you watch *Game of Thrones*?

☒ (e) Some students were late to class.

8. (8 points) Write the negation of each statement in a correct sentence.

(a) He has two dogs.

He DOES NOT HAVE TWO DOGS.

(b) Some rectangles are not squares.

ALL RECTANGLES ARE SQUARES.

(c) Nobody will ever run a 3-minute mile.

SOME ONE WILL RUN A 3-MINUTE MILE.

(d) Someone in this class will get an A.

NO ONE IN THIS CLASS WILL GET AN A.

9. (4 points) Let p = "She gets paid" and let q = "She buys groceries." Write each statement in words.

(a) $p \wedge q$

SHE GETS PAID AND SHE BUYS GROCERIES.

(b) $\sim p \rightarrow \sim q$

IF SHE DOES NOT GET PAID, THEN SHE DOES NOT
BUY GROCERIES.

10. (4 points) Refer to the statements p and q from the problem directly above. Write each statement in symbolic form.

(a) She gets paid, but she does not buy groceries.

$p \wedge \sim q$

(b) She buys groceries if she gets paid.

$p \rightarrow q$

11. (5 points) Write the truth table for the conditional statement $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

12. (12 points) Use a truth table to determine whether the statement is a tautology, a self-contradiction, or neither.

$$(p \wedge q) \wedge \sim q$$

p	q	$p \wedge q$	$\sim q$	$(p \wedge q) \wedge \sim q$
T	T	T	F	F
T	F	F	T	F
F	T	F	F	F
F	F	F	T	F

All F's \Rightarrow

SELF-CONTRADICTION

13. (6 points) Consider the following conditional statement:

$\underbrace{p}_{\text{If Sara learns Python,}} \quad \underbrace{q}_{\text{then she writes programs.}}$

- (a) Write the contrapositive of the statement.

$\sim q \rightarrow \sim p$: IF SARA DOES NOT WRITE programs,
THE SHE DOES NOT LEARN PYTHON.

- (b) Write the converse of the statement.

$q \rightarrow p$: IF SARA WRITES programs, THEN SHE LEARNS PYTHON.

- (c) Write the inverse of the statement.

$\sim p \rightarrow \sim q$: IF SARA DOES NOT LEARN PYTHON,
THEN SHE DOES NOT WRITE programs.

14. (5 points) Sofia deposits \$8500 into an account earning simple interest. After 4 years, the account is worth \$10,727. What was Sofia's simple interest rate? Write your final answer in percent form.

$$I = 10727 - 8500 = 2227$$

$$2227 = (8500)(r)(4)$$

$$\Rightarrow r = \frac{2227}{(8500)(4)} = 0.0655 = \boxed{6.55\%}$$

15. (10 points) Elaine inherited \$172,000 after the death of a distant relative. She deposited the money, all at once, into an account earning 7.25% compounded quarterly.

- (a) How much money is in the account after 25 years?

$$A = 172000 * \left(1 + \frac{0.0725}{4}\right)^{(4*25)}$$

$$= \boxed{\$1,036,665.42}$$

- (b) How much money was made in interest?

$$1036665.42 - 172000$$

$$= \boxed{\$864,665.42}$$

16. (8 points) Suppose you begin depositing monthly payments into an account earning 4.5% compounded monthly. Your goal is to accumulate \$50,000 in 10 years. What should your monthly payments be?

$$R = \frac{50000 \cdot \left(\frac{0.045}{12}\right)}{\left(\left(1 + \frac{0.045}{12}\right)^{(12*10)} - 1\right)} = \boxed{\$330.69}$$

17. (12 points) A house sells for \$335,000 and an 5% down payment is made. The remaining amount is mortgaged for 30 years at a fixed rate of 4.5% compounded monthly.

(a) How much is the down payment?

$$5\% \text{ of } 335000 = 0.05 * 335000 = \$16,750$$

(b) What is the loan amount?

$$335000 - 16750 = \$318,250$$

(c) Determine the monthly mortgage payment on the loan amount.

$$R = \frac{318250 * (0.045/12)}{\left(1 - \left(1 + \frac{0.045}{12}\right)^{-12*30}\right)} = \$1612.53$$

(d) When the loan is paid off in 30 years, what will be the total interest paid?

$$1612.53 * 12 * 30 - 318250 = \$262,260.80$$

18. (6 points) A letter is selected at random from the word *TENNESSEE*.

(a) What is the sample space?

$$\{T, E, N, S\}$$

9 LETTERS,
EQUALLY LIKELY

(b) What is the event of selecting a vowel?

$$\{E\}$$

(c) What is the probability of selecting a *T* or *N*?

$$\frac{3}{9}$$

19. (5 points) The probability of the event A is $7/19$. What are the odds in favor of A ?
What are the odds against A ? (Indicate which answer is which.)

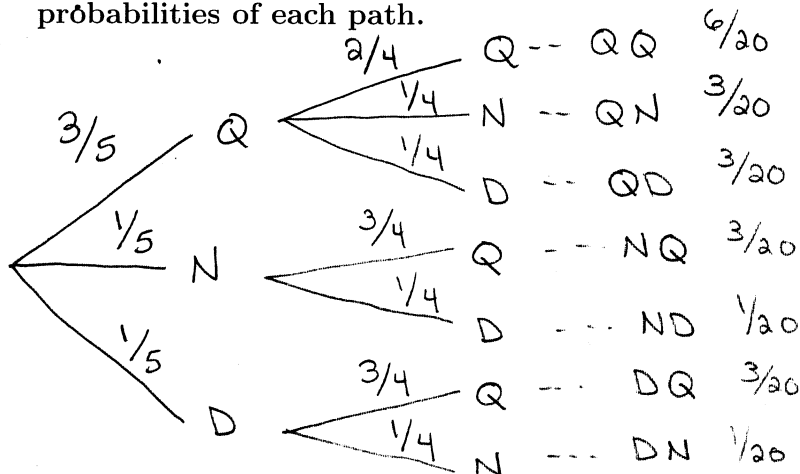
$$\text{Odds} = \frac{7}{19-7} = \boxed{\frac{7}{12}} \quad \text{Odds Against} = \boxed{\frac{12}{7}}$$

20. (5 points) A standard 6-sided die is rolled one time. Compute the expected value.

$$E = 1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right) \\ = \frac{1+2+3+4+5+6}{6} = \frac{21}{6} = \boxed{3.5}$$

21. (10 points) A jar contains 3 quarters, 1 dime, and 1 nickel. Two coins are selected at random (without replacement).

- (a) Sketch the complete tree diagram for this experiment. Include the probabilities of each path.



- (b) What is the probability of selecting 50 cents?

$$\frac{6}{20}$$

- (c) What is the probability of selecting less than 50 cents?

$$1 - \frac{6}{20} = \boxed{\frac{14}{20}}$$