Math 115 - Test 3
November 13, 2014

Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (10 points) Determine whether each statement is true (T) or false (F).

(a) F If A and B are any events, then $P(A \cup B) = P(A) + P(B) \oplus P(A \cap B)$.
$P(A) + P(B) - P(A \cap B)$

(b) T When two events are mutually exclusive, they have no outcomes in common.

(c) T If A and B are independent, then $P(A|B) = P(A)$.

(d) F To get the probability of a path in a tree diagram, you ~~add~~ multiply the probabilities along the branches.

(e) F Whenever two events are independent, they are also mutually exclusive.

(f) F The probability distribution associated with the experiment of rolling a six-sided die is an example of a ~~continuous~~ DISCRETE probability distribution.

(g) F The ~~mean (expected value)~~ STD. DEV. of a random variable of a probability distribution describes how the outcomes vary.

(h) T In most applications, continuous random variables represent measured data, while discrete random variables represent counted data.

(i) F The sum of all the probabilities in a discrete probability distribution must be ~~less than~~ EQUAL TO one.

(j) F If the probability of success in a binomial experiment is $p = 0.64$, then the probability of failure is ~~$q = 0.46$.~~ $q = 0.36$

2. (2 points) On a single roll of a six-sided die, what is the probability of rolling a two given that you rolled an even number? Give a brief explanation.

GIVEN THAT AN EVEN IS ROLLED, THE SAMPLE SPACE IS REDUCED TO $\{2,4,6\}$

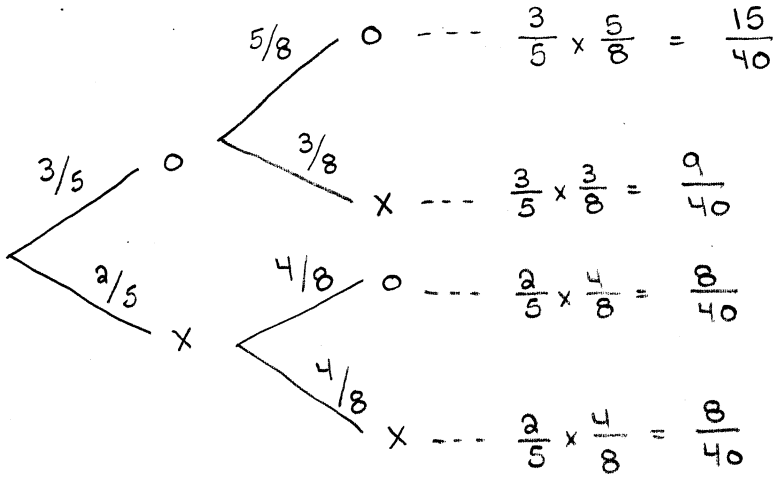
PROB OF ROLLING A 2 IS THEN $\boxed{\frac{1}{3}}$

3. (8 points) A letter is selected at random from the first box and placed into the second box. Then a letter is selected at random from the second box.

o	o	o	x	x

o	o	o	o	x	x	x

(a) Sketch the complete tree diagram for this experiment. **Include the probability of each path.**



(b) What is the probability of selecting at least one letter o?

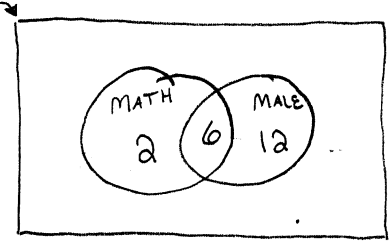
$$\frac{15}{40} + \frac{9}{40} + \frac{8}{40} = \boxed{\frac{32}{40}}$$

(c) What is the probability of selecting the letter x from the second box?

$$\frac{9}{40} + \frac{8}{40} = \boxed{\frac{17}{40}}$$

4. (3 points) A math class has 28 students. Of these, 8 students are math majors and 18 students are males. Of the math majors, 6 are males. Find the probability that a randomly selected student is a male or a math major.

28



$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{2+6+12}{28} = \frac{20}{28}$

- OR -

$\frac{8}{28} + \frac{18}{28} - \frac{6}{28} = \frac{20}{28}$

5. (3 points) Determine whether the random variable x is discrete or continuous.

   (a) Let x represent the volume of Coke in a can of soda.

   CONTINUOUS

   (b) Let x represent the number of dogs in a household.

   DISCRETE

   (c) Let x represent the number of characters in a text message.

   DISCRETE

6. (3 points) Consider the following discrete probability distribution.

x	0	1	2	3	4	5	6
$P(x)$	0.5	?	0.23	0.21	0.17	0.11	0.08

   (a) Find the missing probability.

   $0.5 + 0.23 + 0.21 + 0.17 + 0.11 + 0.08$

   $= 1.3$ THIS WOULD MAKE $P(1) = -0.3$

   (b) Find $P(x > 3)$, that is, the probability that x is greater than 3. NOT A PROB DIST

   ASSUMING THE PROBS ARE CORRECT...

   $0.17 + 0.11 + 0.08 = 0.36$

   YIKES! TYPO SOMEWHERE.

   (c) Find $P(x \neq 1)$, that is, the probability that x is not 1.

   FOUND THIS IN PART (a) AND GOT $P(x \neq 1) = 1.3$

YIKES! CAN'T BE CORRECT! TYPO! ☹

7. (4 points) Determine if the events are independent or dependent. Give a brief, one-sentence explanantion for each.

(a) Selecting a king from a standard deck of playing cards, replacing it, and then selecting a queen from the deck

INDEPENDENT BECAUSE THE SELECTIONS WERE MADE WITH REPLACEMENT. 1ST DRAW DOES NOT AFFECT 2ND

(b) Rolling a six-sided die and then rolling it a second time so that the sum of the two rolls is five

DEPENDENT. TO GET A SUM OF 5, THE 2ND ROLL DEPENDS ON THE 1ST ROLL.

8. (6 points) The probability distribution for the number of games played in the World series from 1903 to 2012 is shown below.

x	4	5	6	7	8
$P(x)$	0.176	0.241	0.213	0.333	0.037

(a) Find the mean (expected value).

$\mu = 4(0.176) + 5(0.241) + 6(0.213) + 7(0.333) + 8(0.037)$

$\mu = 5.814$

(b) Find the standard deviation.

$\sigma^2 = 16(0.176) + 25(0.241) + 36(0.213) + 49(0.333) + 64(0.037) - (5.814)^2$

$= 1.3914\ldots$

$\sigma \approx 1.180$

(c) What is an unusually large number of games in the World Series?

$\mu + 2\sigma \approx 5.814 + 2(1.180)$

$= 8.174$

THIS MAKES NO NUMBER OF GAMES UNUSUALLY LARGE.

9. (5 points) The numbers of students at a certain college are described in the table below.

	Female	Male	
Part-time	2112	1408	3520
Full-time	1746	1164	2910
	3858	2572	6430

A college student is selected at random.

(a) What is the probability that the student is a female?

$$\frac{3858}{6430} = 0.60$$

(b) What is the probability that the student is a part-time student?

$$\frac{3520}{6430} \approx 0.547$$

(c) What is the probability that the student is a female, part-time student?

$$\frac{2112}{6430} \approx 0.328$$

(d) What is the probability that the student is a female student or a part-time student?

$$\frac{2112 + 1746 + 1408}{6430} = \frac{5266}{6430} \approx 0.819$$

(e) What is the probability that the student is a female given that the student is part-time?

$$\frac{2112}{3520} = 0.60$$

10. (6 points) In a certain town, 63% of the children dress as vampires on Halloween. Ten children are randomly selected.

(a) What is the probability that 6 or fewer are dressed as vampires?

BINOMIAL
$p = 0.63$
$q = 0.37$
$N = 10$

$$P(x \leq 6) = \text{binomcdf}(10, 0.63, 6)$$
$$= \boxed{0.5400}$$

(b) What is the probability that more than 6 are dressed as vampires?

$$P(x > 6) = 1 - P(x \leq 6)$$
$$= 1 - 0.5400 = \boxed{0.4600}$$

(c) How many vampires would you expect in a sample of 10?

$$\mu = 10(0.63) = \boxed{6.3}$$

(d) In a sample of 10, what would be an unusually small number of vampires?

$$\mu - 2\sigma = np - 2\sqrt{npq}$$
$$= 6.3 - 2\sqrt{10(0.63)(0.37)}$$
$$\approx \boxed{3.25}$$

3 OR FEWER WOULD BE UNUSUAL