Math 115 - Final Exam

December 11, 2014

Name key Score ________

Show all work to receive full credit. Supply explanations where necessary.

1. (10 points) Determine whether each statement is true (T) or false (F).

   (a) T In a normal distribution, the mean and median are equal.

   (b) F When constructing a confidence interval estimate, choosing a larger sample causes the margin of error to be ~~greater~~ SMALLER.

   (c) F The normal distribution is a ~~discrete~~ CONTINUOUS probability distribution.

   (d) F To get the probability of a path in a tree diagram, you ~~add~~ MULTIPLY the probabilities along the branches.

   (e) F In a frequency distribution, the class width is the distance between ~~the~~ CONSECUTIVE lower ~~and upper~~ limits of a class.

   (f) F The mean is the measure of central tendency that is ~~least~~ GREATLY affected by outliers.

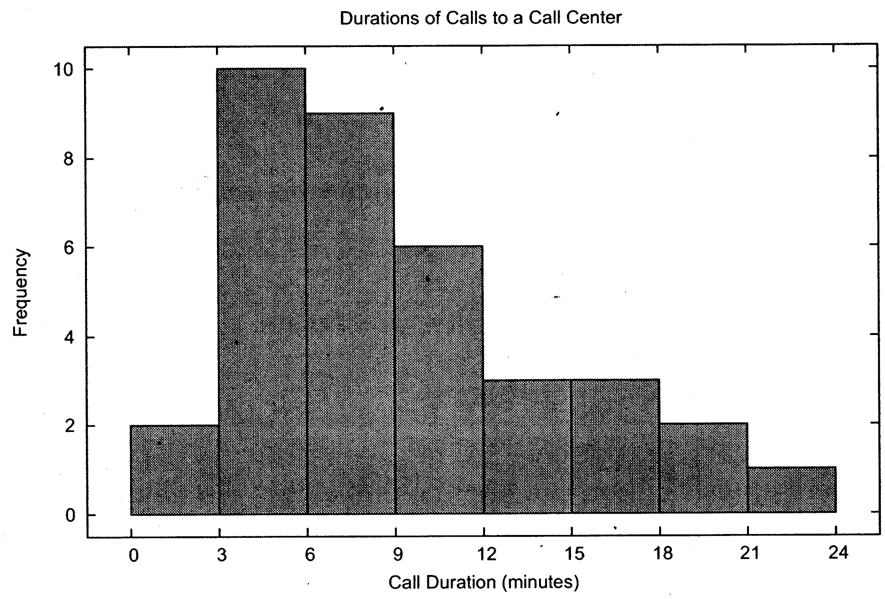
   (g) T The standard deviation of a probability distribution describes how the outcomes vary.

   (h) T In most applications, continuous random variables represent measured data, while discrete random variables represent counted data.

   (i) T In a data set, roughly 50% of data values lie between Q_1 and Q_3.

   (j) F The Poisson distribution is a ~~continuous~~ DISCRETE probability distribution.

2. (4 points) The histogram below shows the distribution of the lengths of phone calls (in minutes) to a customer service center on a particular day.



(a) How many phone calls did the service center receive on that particular day?

$2+10+9+6+3+3+2+1 = \boxed{36}$

(b) Which time interval had the least number of calls?

$\boxed{21 \text{ MIN} - 24 \text{ MIN}}$ HAD ONLY 1 CALL

(c) If the histogram was changed to a relative frequency histogram, what would be the height of the fourth bar?

$\frac{6}{36} = \boxed{\frac{1}{6} = 0.1\overline{6} \approx 17\%}$

(d) Without attempting to compute them, determine which is greater, the mean length of phone calls or the median length?

SINCE THE DISTRIBUTION IS SKEWED RIGHT,

$\boxed{\text{MEAN} > \text{MEDIAN.}}$

3. (6 points) The following frequency distribution shows the costs (in dollars) of 30 portable GPS navigators.

CLASS WIDTH IS 40

GPS Costs ($)	Frequency
65–104	6
105–144	9
145–184	6
185–224	4
225–264	2
265–304	1
305–344	2

30

(a) What are the class boundaries associated with the **first** class listed above?

64.5 AND 104.5

(b) What is the class width?

$105 - 65 = \boxed{40}$

(c) What are the class midpoints?

$\frac{104+65}{2} = 84.5$

84.5, 124.5, 164.5, 204.5, 244.5, 284.5, 324.5

(d) Use class midpoints to estimate the (weighted) mean cost.

$$\bar{X} \approx \frac{6(84.5) + 9(124.5) + 6(164.5) + 4(204.5) + 2(244.5) + 1(284.5) + 2(324.5)}{30}$$

$$= \frac{4855}{30} = 161.8\overline{3}$$

$161.83

4. (4 points) The depths (in inches) at which 10 artifacts were found at an archaeological dig are listed below.

20.7 24.8 30.5 26.2 36.0 34.3 30.3 29.5 27.0 38.5

(a) Find the range of the data set.

$38.5 - 20.7 = \boxed{17.8}$

(b) Use your calculator to compute the mean and sample standard deviation.

$\boxed{\bar{X} = 29.78, \quad S = 5.414}$

(c) Use your results from above to determine what depths would be unusually large.

$\bar{X} + 2S = \boxed{40.608}$ ANY DEPTH GREATER THAN 40.608 IS UNUSUAL.

5. (3 points) Math SAT scores have mean 514 and standard deviation 117, whereas math ACT scores have mean 21.1 and standard deviation 5.3. Compute the coefficient of variation (CV) for each test. Which test has more variation?

SAT: $\frac{117}{514} \approx 22.8\%$ ACT: $\frac{5.3}{21.1} \approx 25.1\%$

ACT HAS GREATER VARIATION

6. (3 points) State whether each probability is theoretical or experimental.

(a) A bag contains 3 red marbles, 5 blue marbles, and 2 green marbles. The probability of randomly selecting a green marble is 2/10.

THEORETICAL

(b) The probability of rolling an even number on a six-sided die is 3/6.

THEORETICAL

(c) After rolling a die 75 times, John got an even number 28 times. The probability of rolling an even number is 28/75.

EXPERIMENTAL

7. (8 points) The numbers below show the points scored by the Chicago Bulls in their 2012–2013 season playoff games (in the order in which they occurred).

~~89~~ ~~90~~ ~~79~~ ~~142~~ ~~91~~ ~~92~~
~~99~~ ~~93~~ ~~78~~ ~~94~~ ~~65~~ ~~91~~

Compute the five-number summary, the interquartile range (IQR), and the cutoff values for outliers.

$\underbrace{65, 78, 79, 89, 90, 91}, \underbrace{91, 92, 93, 94, 99, 142}$

$\frac{79+89}{2} = 84 \qquad \frac{91+91}{2} = 91 \qquad \frac{93+94}{2} = 93.5$

$\boxed{\begin{array}{ll} \text{MIN} = 65 & Q_3 = 93.5 \\ Q_1 = 84 & \text{MAX} = 142 \\ \text{MEDIAN} = 91 & \end{array}}$

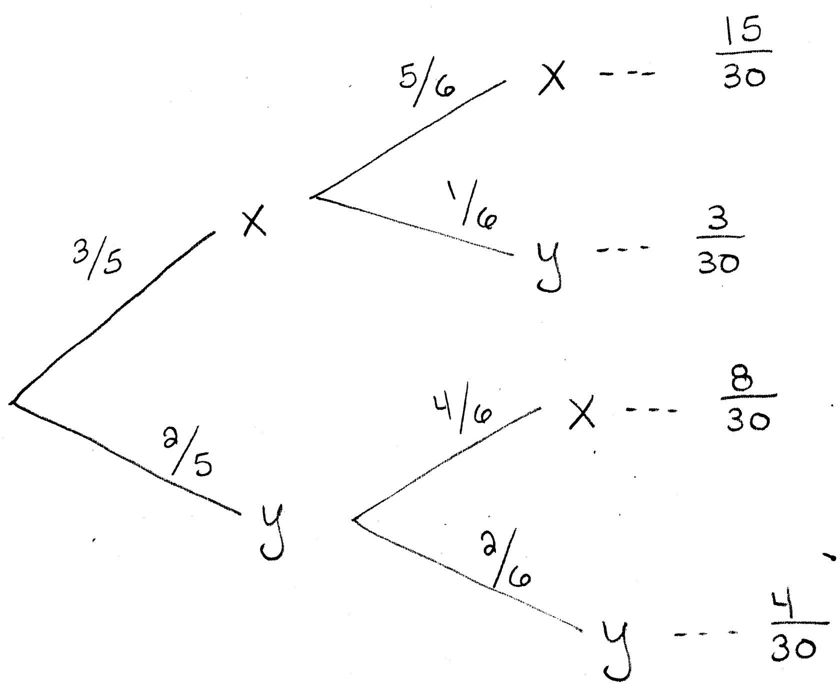
IQR $= 93.5 - 84$
$= 9.5$

OUTLIER CUTOFFS:
$84 - 1.5(9.5)$
$= 69.75$
$93.5 + 1.5(9.5)$
$= 107.75$

8. (5 points) A letter is selected at random from the first box and placed into the second box. Then a letter is selected at random from the second box.

$\boxed{x \quad x \quad x \quad y \quad y} \longrightarrow \boxed{x \quad x \quad x \quad x \quad y}$

Sketch the complete tree diagram for this experiment. Include the probabilities of each path.



9. (3 points) A biology class has 35 students. Of these, 11 students are biology majors and 14 students are male. Of the biology majors, 5 are male. Find the probability that a randomly selected student is a male or a biology major.

B = EVENT OF SELECTING BIO MAJOR

M = EVENT OF SELECTING MALE.

$P(B) = \frac{11}{35}$, $P(M) = \frac{14}{35}$, $P(B \cap M) = \frac{5}{35}$

$$P(B \cup M) = \frac{11}{35} + \frac{14}{35} - \frac{5}{35} = \boxed{\frac{20}{35}}$$

10. (4 points) A card is drawn at random from a standard deck of playing cards. Let A be the event of drawing a face card, and let B be the event of drawing a queen. Compute $P(A|B)$ and $P(B|A)$. Be sure to indicate which is which.

$$\boxed{P(A|B) = \frac{4}{4} = 1}$$

$$\boxed{P(B|A) = \frac{4}{12} = \frac{1}{3}}$$

11. (3 points) In a recent year, SAT scores were normally distributed with mean 1498 and standard deviation 316. What is the probability that a randomly selected test has a score between 1250 and 1450?

normalcdf(1250, 1450, 1498, 316)

$\approx \boxed{0.2234}$

12. (6 points) The probability distribution for the number of games played in the World series from 1903 to 2012 is shown below.

x	4	5	6	7	8
$P(x)$	0.176	0.241	0.213	0.333	0.037

(a) Find the mean (expected value).

$$\mu = 4(0.176) + 5(0.241) + 6(0.213) + 7(0.333) + 8(0.037)$$
$$= \boxed{5.814}$$

(b) Find the standard deviation.

$$\sigma^2 = 16(0.176) + 25(0.241) + 36(0.213) + 49(0.333) + 64(0.037) - 5.814^2$$
$$= 1.391404 \Rightarrow \boxed{\sigma \approx 1.18}$$

(c) What is an unusually large number of games in the World Series?

$$\mu + 2\sigma = 5.814 + 2(1.18) = \boxed{8.174}$$

MORE THAN 8 IS UNUSUAL
(BUT THIS DOES NOT HAPPEN)

13. (6 points) In a certain region, polls indicate that Democrats have a 62% chance of winning elections. Fifteen races are selected at random in that region. (Use the binomial distribution for these problems.)

(a) What is the probability that exactly 10 Democrats win elections?

$N=15$
$p=0.62$
$q=0.38$

$$P(X=10) = \text{binompdf}(15, 0.62, 10)$$
$$\approx \boxed{0.1997}$$

(b) What is the probability that more than 8 Democrats win elections?

$$P(X>8) = 1 - P(X \le 8) = 1 - \text{binomcdf}(15, 0.62, 8)$$
$$\approx \boxed{0.6705}$$

(c) What would be an unusually small number of winning Democrats?

$\mu = Np = 9.3$

$\sigma = \sqrt{Npq} = 1.88$

$$9.3 - 2(1.88) = \boxed{5.54}$$

5 OR FEWER WOULD BE UNUSUAL

14. (4 points) Heights of U.S. women aged 20–29 are normally distributed with mean 64.2 in and standard deviation 2.9 in. In a group of 750 women, about how many are taller than 68 in?

750 × normalcdf(68, 99999, 64.2, 2.9)

$\approx \boxed{71.28}$

About 71

15. (3 points) In a survey of 3110 U.S. adults, 1435 say they have started paying some bills online. Find a 90% confidence interval estimate for the true population proportion. Give a one-sentence interpretation of your result.

1-PropZInt

X = 1435

n = 3110

C-Level = 0.90

(0.44671, 0.47612)

WE CAN BE 90% CONFIDENT THAT THE TRUE POP. PROPORTION IS BETWEEN 44.7% AND 47.6%.

16. (3 points) The waiting times at the emergency room of a certain hospital and approximately normally distributed. In a sample of 21 patients, the mean waiting time was 24 minutes and the sample standard deviation was 10 minutes. Find a 95% confidence interval estimate for the true population mean. Give a one-sentence interpretation of your result.

TInterval WITH STATS

$\bar{X} = 24$

$S_X = 10$

n = 21

C-Level = 0.95

(19.448, 28.552)

WE CAN BE 95% CONFIDENT THAT THE TRUE POP. MEAN IS BETWEEN 19.4 MIN AND 28.6 MIN.