Math 129 - Review 3

November 11, 2019

Name key

These problems may help you review for Test 3. They are coded to match the course objectives from your syllabus. Your actual test will not be as long as this review packet. Unless otherwise indicated, you should simplify all answers by reducing fractions, simplifying radicals, and/or rationalizing denominators (as you've done on your ALEKS homework). Label your axes when graphing.

Objective: Evaluate difference quotients. [5]

1. Evaluate the difference quotient $\dfrac{g(x+h)-g(x)}{h}$ for the function $g(x) = 4x+1$. Completely expand and simplify your answer.

$$\frac{[4(x+h)+1]-[4x+1]}{h} = \frac{(4x+4h+1)-(4x+1)}{h} = \frac{4h}{h} = \boxed{4}$$

2. Evaluate the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ for the function $f(x) = x^2+x+2$. Completely expand and simplify your answer.

$$\frac{[(x+h)^2+(x+h)+2]-[x^2+x+2]}{h} = \frac{(x^2+2xh+h^2+x+h+2)-(x^2+x+2)}{h}$$

$$= \frac{2xh+h^2+h}{h} = \frac{h(2x+h+1)}{h} = \boxed{2x+h+1}$$

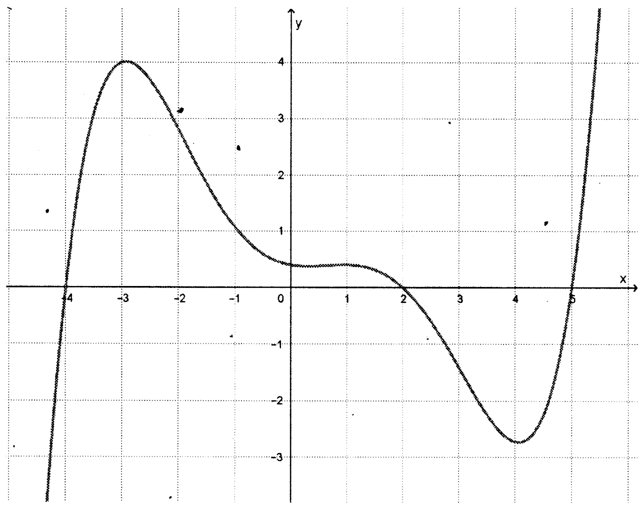
3. Evaluate the difference quotient $\dfrac{g(x+h)-g(x)}{h}$ for the function $g(x) = 2x^2+6$. Completely expand and simplify your answer.

$$\frac{[2(x+h)^2+6]-[2x^2+6]}{h} = \frac{(2x^2+4xh+2h^2+6)-(2x^2+6)}{h}$$

$$= \frac{4xh+2h^2}{h} = \frac{h(4x+2h)}{h} = \boxed{4x+2h}$$

Objective: Given the graph of a function, determine where the function is positive, negative, or zero. [8,10]

4. The graph of the function f is shown below.

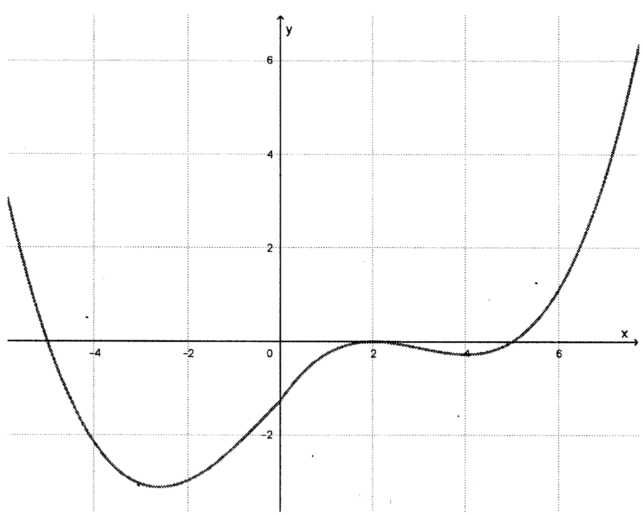


(a) Determine the intervals on which $f(x) > 0$. ABOVE X-AXIS : $(-4,2) \cup (5,\infty)$

(b) Determine the intervals on which $f(x) \leq 0$. BELOW OR ON : $(-\infty,-4] \cup [2,5]$

(c) Determine any x-values for which $f(x) = 0$. $X = -4,\ X = 2,\ X = 5$

5. The graph of the function f is shown below.



(a) Determine the intervals on which $f(x) < 0$. BELOW X-AXIS : $(-5,2) \cup (2,5)$

(b) Determine the intervals on which $f(x) \geq 0$. ABOVE OR ON : $(-\infty,-5] \cup \{2\} \cup [5,\infty)$

(c) Determine any x-values for which $f(x) = 0$. $X = -5,\ X = 2,\ X = 5$

Objective: Given the graph of a function, determine intervals on which the function is increasing, decreasing, or constant. Given the graph of a function, determine the local maxima and minima.[8,10]

6. Look back at the graph of the function in problem 4. Using the graph of f,

   (a) determine intervals on which f is increasing.

   GRAPH MOVING UP TO RIGHT ... $(-\infty, -2.9) \cup (0, 1.5) \cup (4.1, \infty)$

   (b) determine intervals on which f is decreasing.

   GRAPH MOVING DOWN TO RIGHT ...

   $(-2.9, 0) \cup (1.5, 4.1)$

   (c) determine all local (relative) maximum and minimum values of f.

REL. MAX	REL. MIN
$y = 4$ AT $x = -2.9$	$y \approx 0.4$ AT $x = 0$
$y \approx 0.5$ AT $x = 1.5$	$y \approx -2.7$ AT $x = 4.1$

7. Look back at the graph of the function in problem 5. Using the graph of f,

   (a) determine intervals on which f is increasing.

   UP TO THE RIGHT ... $(-2.5, 2) \cup (4.1, \infty)$

   (b) determine intervals on which f is decreasing.

   DOWN TO THE RIGHT ... $(-\infty, -2.5) \cup (2, 4.1)$

   (c) determine all local (relative) maximum and minimum values of f.

REL MAX	REL MIN
$y = 0$ AT $x = 2$	$y = -3$ AT $x = -2.5$
	$y = -0.25$ AT $x = 4.1$

Objective: Determine if a graph has symmetry. [8,10]

8. Given a function f, how do we determine algebraically if its graph is symmetric about the y-axis?

SYMMETRY ABOUT Y-AXIS : $f(-x) = f(x)$
(EVEN SYMMETRY)

9. Without sketching the graph, show algebraically that the graph of $f(x) = 3x^2 + 1$ is symmetric about the y-axis.

$$f(-x) = 3(-x)^2 + 1 = 3x^2 + 1 = f(x)$$

10. Given a function f, how do we determine algebraically if its graph is symmetric about the origin?

SYMMETRY ABOUT THE ORIGIN : $f(-x) = -f(x)$
(ODD SYMMETRY)

11. Without sketching the graph, show algebraically that the graph of $f(x) = x^3 + 2x$ is symmetric about the origin.

$$f(-x) = (-x)^3 + 2(-x) = -x^3 - 2x = -f(x)$$

12. What symmetry does the graph of $y = 1/x$ exhibit?

GRAPH IS SYMMETRIC ABOUT THE ORIGIN.
(ODD SYMMETRY)

13. What symmetry does the graph of $y = 1/x^2$ exhibit?

GRAPH IS SYMMETRIC ABOUT Y-AXIS.
(EVEN SYMMETRY)

Objective: Develop a familiarity with the graphs of basic functions (Toolbox Functions). [8,10]

14. For each of our basic toolbox functions (constant, linear, absolute value, squaring, cubing, square root, cube root, reciprocal, and reciprocal square functions) know each of the following:

    (a) shape of the graph,

    (b) domain and range,

    (c) any symmetry in the graph, and

    (d) any vertical or horizontal asymptotes of the graph.

SEE NOTES OR TEXTBOOK. (OR ASK ME.)

Objective: Apply the transformations (shifts, reflections, stretches, and compressions) to basic graphs to obtain more general graphs. [9]

15. What are the domain and range of $f(x) = \sqrt{x - 16} + 9$?

16 RIGHT, 9 UP

DOMAIN OF $y = \sqrt{x}$ IS $[0, \infty)$

RANGE OF $y = \sqrt{x}$ IS $[0, \infty)$

$\Rightarrow$

$$\boxed{\begin{array}{l}\text{DOMAIN OF } f: [16, \infty) \\ \text{RANGE OF } f: [9, \infty)\end{array}}$$

16. The graph of $y = (x + 5)^2 - 8$ is a parabola. Where is its vertex?

GRAPH OF $y = x^2$ HAS VERTEX AT $(0,0)$

5 LEFT, 8 DOWN

$$\boxed{\text{VERTEX AT } (-5, -8)}$$

17. What are the domain and range of $g(x) = |x + 8| + 5$?

LEFT 8, UP 5

DOMAIN OF $y = |x|$ IS $(-\infty, \infty)$

RANGE OF $y = |x|$ IS $[0, \infty)$

$\Rightarrow$

$$\boxed{\begin{array}{l}\text{DOMAIN OF } g: (-\infty, \infty) \\ \text{RANGE OF } g: [5, \infty)\end{array}}$$

18. **Carefully sketch** the graph of each function. Your graph should show details such as correct scale and position. (Label your axes.)

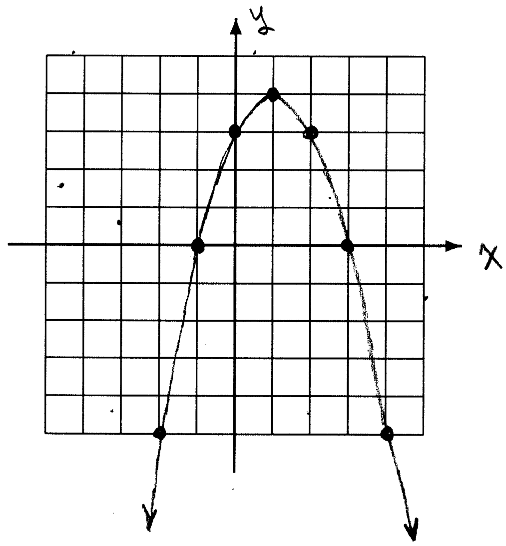
(a) $g(x) = 4 - (x-1)^2$

$y = x^2$

① Right 1

② Reflect about x-axis

③ Up 4



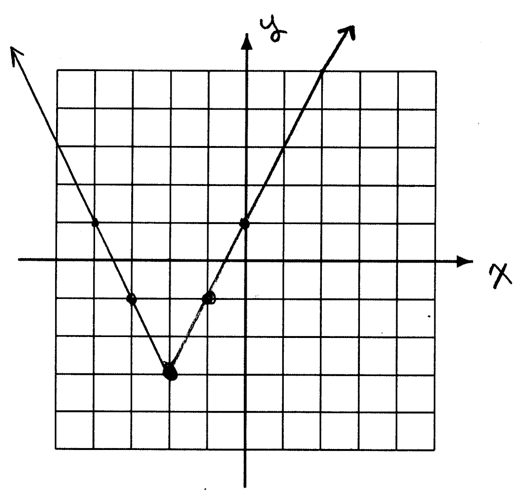
(b) $f(x) = 2|x+2| - 3$

$y = |x|$

① Left 2

② Stretch x 2

③ Down 3



Objective: Determine the transformations that result in a given graph. [9]

19. Describe the sequence of transformations that transform the graph of $y = x$ to that of $y = (x + 4) - 5$.

① SHIFT LEFT 4 UNITS

② SHIFT DOWN 5 UNITS

20. Describe the sequence of transformations that transform the graph of $y = x^3$ to that of $y = -(x - 2)^3 + 8$.

① SHIFT RIGHT 2 UNITS

② REFLECT ABOUT X-AXIS

③ SHIFT UP 8 UNITS

21. How is the graph of $y = \sqrt{x}$ related to the graph of $y = 5\sqrt{x}$?

VERTICALLY STRETCHED BY A FACTOR OF 5.

EVERY NEW y-COORD IS 5 TIMES THE OLD y-COORD.

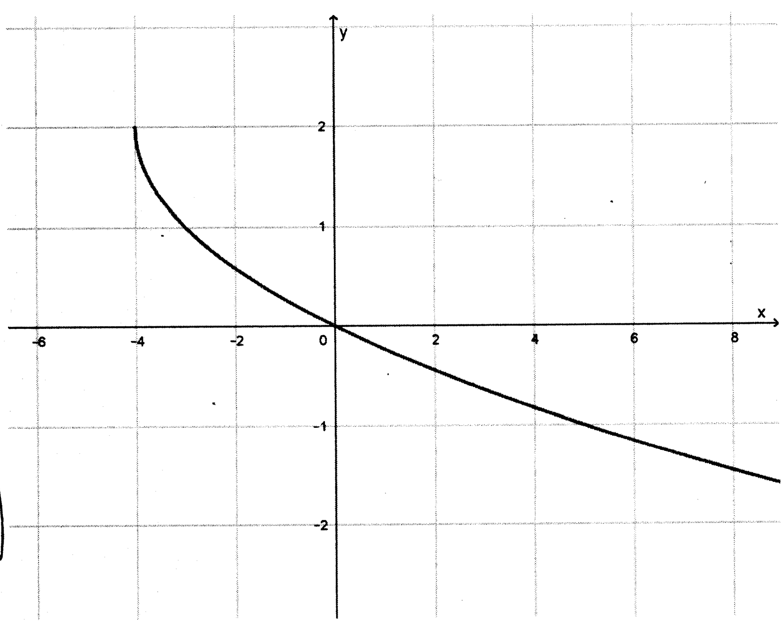
22. The graph of $y = \sqrt{x}$ is transformed by shifts and reflections to obtain the new graph shown below. What is an equation for the new graph?

① LEFT 4

② REFLECT ABOUT X-AXIS

③ UP 2

$\boxed{y = -\sqrt{x+4} + 2}$



Objective: Develop a familiarity with basic rational functions. [10,13]

23. Describe the sequence of transformations that transform the graph of $y = \frac{1}{x}$ to that of $y = \frac{2}{x} + 10$.

① VERTICAL STRETCH BY A FACTOR OF 2

② SHIFT UP 10 UNITS

24. Determine the horizontal and vertical asymptotes of the graph of $y = \frac{15}{x+167} - 293$.

$\boxed{\begin{array}{ll} \text{H.A.} & y = -293 \\ \text{V.A.} & x = -167 \end{array}}$

$y = \frac{1}{x}$ H.A. $y = 0$

V.A. $x = 0$

① LEFT 167

② STRETCH BY FACTOR OF 15

③ DOWN 293

25. Determine the horizontal and vertical asymptotes of the graph of $y = \frac{1}{(x+7)^2} + 8$.

$\boxed{\begin{array}{ll} \text{H.A.} & y = 8 \\ \text{V.A.} & x = -7 \end{array}}$

$y = \frac{1}{x^2}$ H.A. $y = 0$

V.A. $x = 0$

① LEFT 7

② UP 8

Objective: Define and evaluate piecewise functions. [1,5]

26. Consider the function

$$f(x) = \begin{cases} 8, & x < -1 \\ x^2, & -1 \leq x < 1 \\ \sqrt{x}, & x > 1 \end{cases}$$

(a) Evaluate $f(5)$.

3RD PIECE ... $f(5) = \boxed{\sqrt{5}}$

(b) Evaluate $f(-6)$.

1ST PIECE ... $f(-6) = \boxed{8}$

(c) Evaluate $f(1)$

$f(1)$, NO PIECE ... $f(1)$ IS $\boxed{\text{NOT DEFINED.}}$

(d) What is the domain of f?

$(-\infty, 1) \cup (1, \infty)$

(e) Is f a continuous function?

NO, IT IS NOT EVEN DEFINED AT $X=1$. AND THE PIECE ENDING BEFORE $X=-1$ DOES NOT MATCH THE PIECE STARTING AT $X=-1$. $8 \neq 1$

27. Consider the function

$$f(x) = \begin{cases} x^3 - x + 2, & x < 0 \\ x + 6, & x \geq 1 \end{cases}$$

(a) Evaluate $f(-2)$.

1ST PIECE ... $f(-2) = (-2)^3 - (-2) + 2 = -8 + 2 + 2 = \boxed{-4}$

(b) Evaluate $f(1)$.

2ND PIECE ... $f(1) = \boxed{7}$

(c) Evaluate $f(0)$

$f(0)$, NO PIECE ... $f(0)$ IS $\boxed{\text{NOT DEFINED.}}$

(d) What is the domain of f?

$(-\infty, 0) \cup [1, \infty)$

Objective: Sketch the graph of a piecewise-defined function. [1,5]

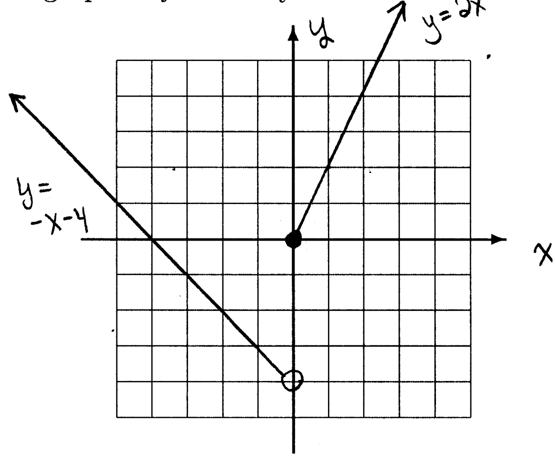
28. Consider the function

$$f(x) = \begin{cases} -x-4, & x < 0 \\ 2x, & x \geq 0 \end{cases}$$

(a) What is the domain of f?

$(-\infty, \infty) = \mathbb{R}$

(b) Carefully sketch the graph of f. Label your axes.

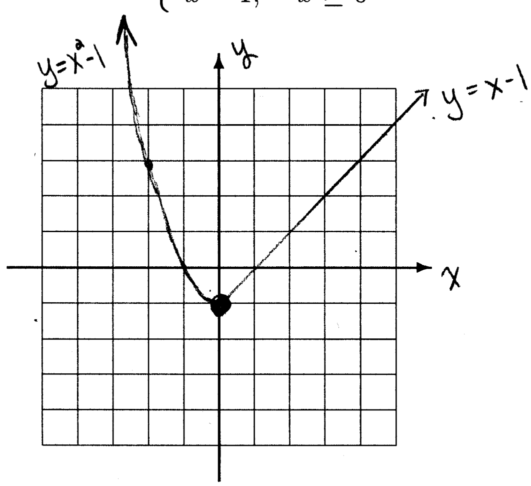


(c) Is f a continuous function? If not, where is it discontinuous?

No, DISCONTINUOUS AT $x = 0$ (GRAPH IS BROKEN.)

29. Sketch the graph of f. Label your axes.

$$f(x) = \begin{cases} x^2-1, & x < 0 \\ x-1, & x \geq 0 \end{cases}$$



Objective: Compute sums, differences, products, and quotients of functions. [5]

30. Let $f(x) = 3x^2 - 8x + 1$ and $g(x) = x^2 + 8x - \sqrt{x}$.

    (a) Find and simplify a formula for $(f+g)(x)$.

    $$(f+g)(x) = (3x^2 - 8x + 1) + (x^2 + 8x - \sqrt{x})$$
    $$= \boxed{4x^2 + 1 - \sqrt{x}}$$

    (b) Evaluate $(f+g)(4)$.

    $$(f+g)(4) = 4(4)^2 + 1 - \sqrt{4} = 64 + 1 - 2 = \boxed{63}$$

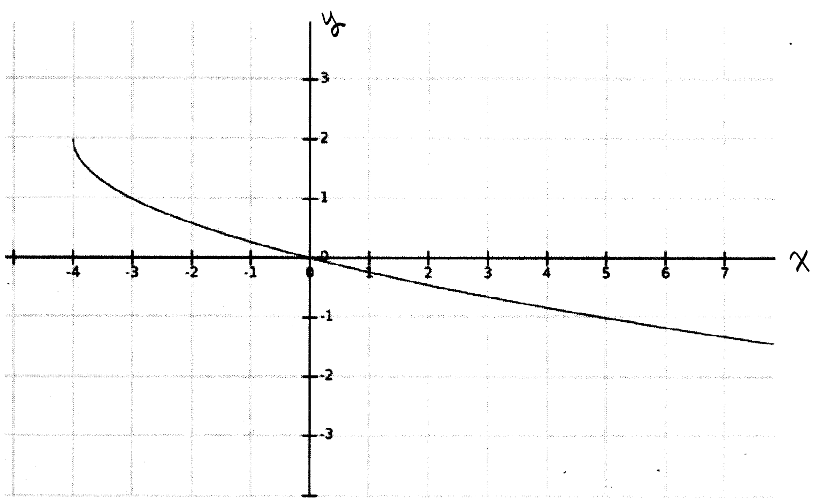
    (c) Determine the domain of $f+g$.

    $$(f+g)(x) = 4x^2 + 1 - \sqrt{x}$$

    $x \geq 0$

    Domain = $[0, \infty)$

31. Let $g(x) = 3x - 5$. Using the function g and the graph of $y = f(x)$ shown below, compute each of the following.



(a) $(g-f)(5) = g(5) - f(5) = [3(5) - 5] - (-1) = 10 + 1 = \boxed{11}$

(b) $(f+g)(-3) = f(-3) + g(-3) = 1 + [3(-3) - 5] = 1 - 14 = \boxed{-13}$

32. Some values of the functions f and g are given in the table below. Use the data from the table to evaluate each of the following.

x	0	1	2	3	4
$f(x)$	2	4	7	1	0
$g(x)$	5	0	4	8	3

(a) $(g - f)(4)$ $\quad g(4) - f(4) = 3 - 0 = \boxed{3}$

(b) $(fg)(3)$ $\quad f(3) \times g(3) = (1)(8) = \boxed{8}$

(c) $\left(\frac{g}{f}\right)(1)$ $\quad \frac{g(1)}{f(1)} = \frac{0}{4} = \boxed{0}$

(d) $(f + f)(4)$ $\quad f(4) + f(4) = 2f(4) = 2(0) = \boxed{0}$

(e) $\left(\frac{f}{g}\right)(1)$ $\quad \frac{f(1)}{g(1)} = \frac{4}{0}$ $\boxed{\text{NOT DEFINED}}$

Objective: Compute a composition of functions. [5]

33. Refer to the functions f and g defined in the problem above.

(a) What is the domain of $(g \circ f)$?

$\{0, 1, 3, 4\}$

(b) What is the range of $(g \circ f)$?

$\{4, 3, 0, 5\}$

34. Let $f(x) = \sqrt{x}$ and $g(x) = 2x + 1$. Evaluate each of the following.

(a) $(f \circ g)(4) = f(g(4)) = f(9) = \boxed{3}$

(b) $(g \circ f)(9) = g(f(9)) = 2(3) + 1 = \boxed{7}$

35. Let $f(x) = \sqrt{2x-4}$ and $g(x) = 3x+5$.

(a) Find and simplify the formula for $(f \circ g)(x)$.

$$f(g(x)) = f(3x+5) = \sqrt{2(3x+5)-4} = \sqrt{6x+10-4}$$
$$= \boxed{\sqrt{6x+6}}$$

(b) What is the domain of $(f \circ g)$?

$$6x+6 \geq 0$$
$$\Rightarrow 6x \geq -6 \Rightarrow x \geq -1$$
$$\boxed{\text{DOMAIN} = [-1, \infty)}$$

Objective: Write a function as a composition of functions. [5]

36. Find two functions f and g so that $(f \circ g)(x) = \sqrt{x^2+x+1}$.

$$\boxed{\begin{aligned} f(x) &= \sqrt{x} \\ g(x) &= x^2+x+1 \end{aligned}}$$

37. Find two functions f and g so that $(f \circ g)(x) = (2x+1)^5 + 7(2x+1)^3$.

$$\boxed{\begin{aligned} f(x) &= x^5 + 7x^3 \\ g(x) &= 2x+1 \end{aligned}}$$

Objective: Solve problems involving operations on functions. [5]

38. An object is cooling in such a way that its temperature in degrees Celsius after t minutes is given by $C = 80 - 0.01t^2$. The formula $f = \frac{9}{5}c + 32$ is used to convert from temperatures in Celsius to Fahrenheit. Find the temperature of the object in degrees Fahrenheit at time t.

$$T(t) = (f \circ C)(t)$$
$$= \frac{9}{5}(80 - 0.01t^2) + 32 = 176 - 0.018t^2$$
$$\boxed{T(t) = 176 - 0.018t^2}$$