Math 129 - Review 4

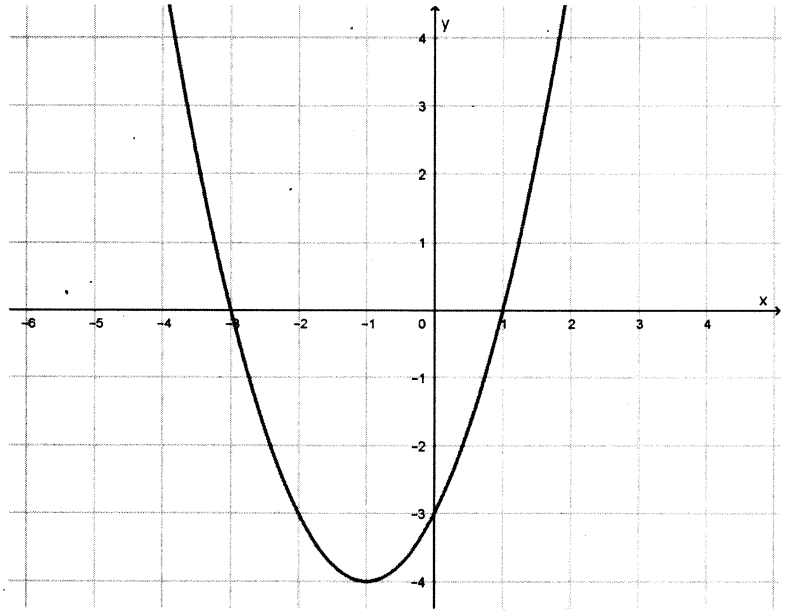
December 8, 2019

Name key

These problems may help you review for the final exam. They are coded to match the course objectives from your syllabus. Unless otherwise indicated, you should simplify all answers by reducing fractions, simplifying radicals, and/or rationalizing denominators (as you've done on your ALEKS homework). Label your axes when graphing. **To adequately prepare for the comprehensive final exam, you should also study the earlier review packets, as well as old tests.**

Objective: Find the vertex, intercepts, and symmetry axis of a parabola. [8]

1. The graph of the function f is the parabola shown below.



(a) Is the leading coefficient of $f(x)$ positive or negative? How do you know?

Positive, because the parabola opens upward.

(b) Determine the vertex of the parabola.

$(-1,-4)$

(c) Determine the x-intercepts of the graph. Write them as ordered pairs.

$(1,0)$ & $(-3,0)$

(d) Determine the y-intercept of the graph. Write it as an ordered pair.

$(0,-3)$

(e) Write an equation for the axis of symmetry of the parabola.

$x = -1$

2. The graph of $y = (x+5)^2 - 8$ is a parabola. Where is its vertex? What is an equation of the symmetry axis?

Vertex at $(-5, 8)$

Axis of symmetry: $x = -5$

3. A quadratic function has a leading coefficient of 3 and zeros $x = 6$ and $x = -2$. Write its equation in standard form. What are the coordinates of the vertex of its graph?

$f(x) = 3(x-6)(x+2)$

$= 3x^2 - 12x - 36$

Vertex... $x = \frac{6+(-2)}{2} = 2$

$f(2) = 3(-4)(4) = -48$

$(2, -48)$

4. The graph of $f(x) = -2(x-3)^2 + 4$ is a parabola.

(a) Does the parabola open up or down? How do you know?

Down. $a = -2 < 0$

(b) Determine the vertex of the parabola.

$(3, 4)$

(c) Write an equation for the symmetry axis.

$x = 3$

(d) Determine the x-intercepts of the parabola.

$-2(x-3)^2 + 4 = 0$

$\Rightarrow (x-3)^2 = 2$

$x - 3 = \pm\sqrt{2}$

$x = 3 \pm \sqrt{2}$

$(3+\sqrt{2}, 0)$ & $(3-\sqrt{2}, 0)$

Objective: Write a quadratic function in vertex form. [6,8]

5. Write the quadratic function $f(x) = x^2 + 6x + 11$ in vertex form.

$f(x) = x^2 + 6x + 9 + 2$

$f(x) = (x+3)^2 + 2$

6. Write the quadratic function $g(x) = x^2 - 4x - 2$ in vertex form.

$g(x) = x^2 - 4x + 4 - 6$

$g(x) = (x-2)^2 - 6$

7. Write the quadratic function $h(x) = 2x^2 + 16x - 24$ in vertex form.

$h(x) = 2(x^2 + 8x - 12)$

$= 2(x^2 + 8x + 16 - 28)$

$h(x) = 2(x+4)^2 - 56$

Objective: Solve application problems involving quadratic functions and parabolas. [8]

8. A ball is thrown upward with a velocity of 48 feet per second from the top of a 144-foot building. Its height, in feet, at time t (in seconds) is given by $h(t) = -16t^2 + 48t + 144$. What is the maximum height of the ball? How long until the ball hits the ground?

Vertex 2

$t = \frac{-48}{-32} = \frac{3}{2} = 1.5$

$h(1.5) = 180$ FT

$h(t) = 0$

$-16t^2 + 48t + 144 = 0$

$t^2 - 3t - 9 = 0$

$t = \frac{3 + \sqrt{9 - 4(1)(-9)}}{2} = \frac{3 + \sqrt{45}}{2}$

≈ 4.85 sec

9. The total cost of manufacturing a set of golf clubs is given by

$$C(x) = 800 - 10x + 0.20x^2,$$

where x is the number of sets of golf clubs produced. How many sets of golf clubs should be manufactured to incur minimum cost and what is that minimum cost?

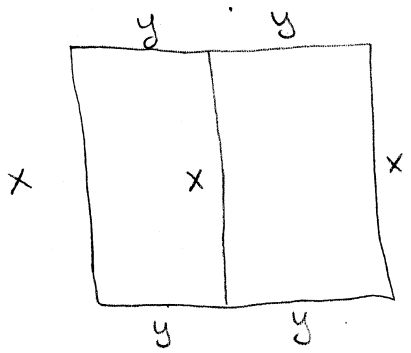
VERTEX ?

$$X = \frac{-b}{2a} = \frac{10}{2(0.2)} = 25$$

$$C(25) = 675$$

MIN COST IS $675 WHEN X = 25

10. Cindy wants to construct two side-by-side dog-training pens of the same size and sharing one common side. She has 400 ft of fencing material to use. What values of x and y maximize the combined areas of the pens?



$3x + 4y = 400$ OR $y = 100 - \frac{3}{4}x$

$$\text{AREA} = x(2y) = 2x\left(100 - \frac{3}{4}x\right)$$

$$= 200x - \frac{3}{2}x^2$$

VERTEX ?

$$x = \frac{-b}{2a} = \frac{200}{3} \text{ FT}$$

$$y = 100 - \frac{3}{4}\left(\frac{200}{3}\right) = 50 \text{ FT}$$

Objective: Find the zeros of a polynomial and determine their multiplicities. [12]

11. Find the zeros of $f(x) = x^2 + 3x + 2$.

$$f(x) = (x+2)(x+1) = 0$$

$x = -2, \quad x = -1$

12. Find the zeros of $g(x) = 4(x-1)^3(x+5)(x+8)^2$ and state their multiplicities.

$x = 1$, mult 3

$x = -5$, mult 1

$x = -8$, mult 2

13. Let $f(x) = x^7(x-1)^5(x+2)$. Find the zeros of f and state their multiplicities.

$x = 0$, mult 7

$x = 1$, mult 5

$x = -2$, mult 1

14. Let $p(x) = x^5 - 4x^4 + 4x^3$. Find the zeros of p and state their multiplicities.

$p(x) = x^3(x^2 - 4x + 4)$

$= x^3(x-2)^2$

$x = 0$, mult 3

$x = 2$, mult 2

Objective: Carry out polynomial long division and synthetic division. [12]

15. Use long division to divide: $\dfrac{3x^2 + 3x - 14}{x - 2}$

$$
\begin{array}{r}
3x + 9 \\
x - 2 \overline{)\,3x^2 + 3x - 14} \\
-(3x^2 - 6x) \\
\hline
9x - 14 \\
-(9x - 18) \\
\hline
4
\end{array}
$$

$3x + 9 + \dfrac{4}{x-2}$

16. Use long division to divide: $(6x^3 - 5x^2 - 3) \div (3x + 2)$

$$
\begin{array}{r}
2x^2 - 3x + 2 \\
3x+2 \overline{)\,6x^3 - 5x^2 + 0x - 3} \\
-(3x^3 + 4x^2) \\
\hline
-9x^2 + 0x - 3 \\
-(-9x^2 - 6x) \\
\hline
6x - 3 \\
-(6x + 4) \\
\hline
-7
\end{array}
$$

$2x^2 - 3x + 2 + \dfrac{-7}{3x+2}$

17. Use long division to divide: $(8x^3 - 6x^2 - 11x + 13) \div (2x^2 - x)$

$$
\begin{array}{r}
4x - 1 \\
2x^2 - x \overline{)\,8x^3 - 6x^2 - 11x + 13} \\
-(8x^3 - 4x^2) \\
\hline
-2x^2 - 11x + 13 \\
-(-2x^2 + x) \\
\hline
-12x + 13
\end{array}
$$

$4x - 2 + \dfrac{-12x + 13}{2x^2 - x}$

18. Use long division to divide: $(-11 + 12x^2 + 5x + 9x^3) \div (-3x^2 - 2x + 2)$

$$
\begin{array}{r}
-3x - 2 \\
-3x^2 - 2x + 2 \overline{)\,9x^3 + 12x^2 + 5x - 11} \\
-(9x^3 + 6x^2 - 6x) \\
\hline
6x^2 + 11x - 11 \\
-(6x^2 + 4x - 4) \\
\hline
7x - 7
\end{array}
$$

$-3x - 2 + \dfrac{7x - 7}{-3x^2 - 2x + 2}$

19. Use synthetic division to divide: $\dfrac{3x^2+3x-14}{x-2}$

$$\begin{array}{r|rr|r} 2 & 3 & 3 & -14 \\ & + & 6 & 18 \\ \hline & 3 & 9 & 4 \end{array}$$

$3x + 9 + \dfrac{4}{x-2}$

20. Use synthetic division to divide: $(3x^4+7x^3-5x^2+x-6) \div (x+3)$

$$\begin{array}{r|rrrr|r} -3 & 3 & 7 & -5 & 1 & -6 \\ & + & -9 & 6 & -3 & 6 \\ \hline & 3 & -2 & 1 & -2 & 0 \end{array}$$

$3x^3 - 2x^2 + x - 2$

21. Use synthetic division to divide: $\dfrac{x^3-21x+20}{x+5}$

$$\begin{array}{r|rrrr} -5 & 1 & 0 & -21 & 20 \\ & + & -5 & 25 & -20 \\ \hline & 1 & -5 & 4 & 0 \end{array}$$

$x^2 - 5x + 4$

Objective: Apply the remainder and factor theorems. [12]

22. Use synthetic division and the remainder theorem to evaluate $P(-2)$ if $P(x) = -2x^4 - 2x^3 + x + 3$.

$$\begin{array}{r|rrrrr} -2 & -2 & -2 & 0 & 1 & 3 \\ & + & 4 & -4 & 8 & -18 \\ \hline & -2 & 2 & -4 & 9 & -15 \end{array}$$

$P(-2) = -15$

23. Use synthetic division and the remainder theorem to evaluate $f(2)$ if $f(x) = 2x^2+3x+1$.

$$\begin{array}{r|rrr} 2 & 2 & 3 & 1 \\ & + & 4 & 14 \\ \hline & 2 & 7 & 15 \end{array}$$

$f(2) = 15$

24. Let $f(x) = x^3 - 2x^2 - 4x + 8$. Evaluate $f(2)$. Based on the value of $f(2)$, determine whether $x - 2$ is a factor of f.

$f(2) = 8 - 8 - 8 + 8 = 0 \Rightarrow$ $x-2$ IS A FACTOR

25. Let $f(x) = 4x^3-3x^2-2x+6$. Evaluate $f(-1)$. Based on the value of $f(-1)$, determine whether $x + 1$ is a factor of f.

$f(-1) = -4 - 3 + 2 + 6 = 1 \Rightarrow$ $x+1$ IS NOT A FACTOR.

26. The only zeros of a polynomial are $x = 0$, $x = 5$ and $x = -8$. Determine the factors of the polynomial.

FACTORS ARE x, $x-5$, AND $x+8$.

Objective: Determine the end behavior of a polynomial function. [12,13]

27. Describe the end behavior of the graph of $f(x) = -4x^8 - 19x^5 + 52x^2 - 17x + 100$.

EVEN DEGREE (8)

WITH NEG. LEADING COEFF.

⇓

DOWN LEFT / DOWN RIGHT

28. Describe the end behavior of the graph of $f(x) = x^3 - 3x^2 - 9x - 17$.

ODD DEGREE (3)

WITH POS. LEADING COEFF.

⇓

DOWN LEFT / UP RIGHT

29. Describe the end behavior of the graph of $f(x) = -3x^2(x+1)(x^2+1)$.

LEADING TERM IS $-3x^5$

ODD DEGREE (5)

WITH NEG. LEADING COEFF.

⇓

UP LEFT / DOWN RIGHT

Objective: Use multiplicities of zeros and end behavior to graph a polynomial function. [12,13]

30. Consider the polynomial $f(x) = -x(x-2)^3(2x+1)^2$.

(a) Determine the degree of f and the leading coefficient.

LEADING TERM IS $-x(x)^3(2x)^2 = -4x^6$

LEADING COEFF IS -4 ; DEGREE IS 6

(b) State the zeros of f and their corresponding multiplicities.

$x = 0$, MULT 1

$x = 2$, MULT 3

$x = -\frac{1}{2}$, MULT 2

(c) Describe the end behavior of the graph of f.

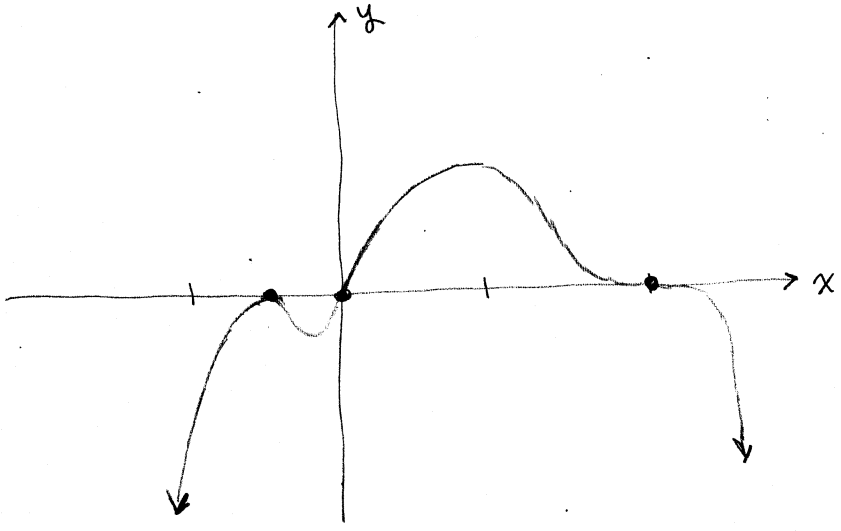
DOWN LEFT / DOWN RIGHT

(d) Determine the y-intercept.

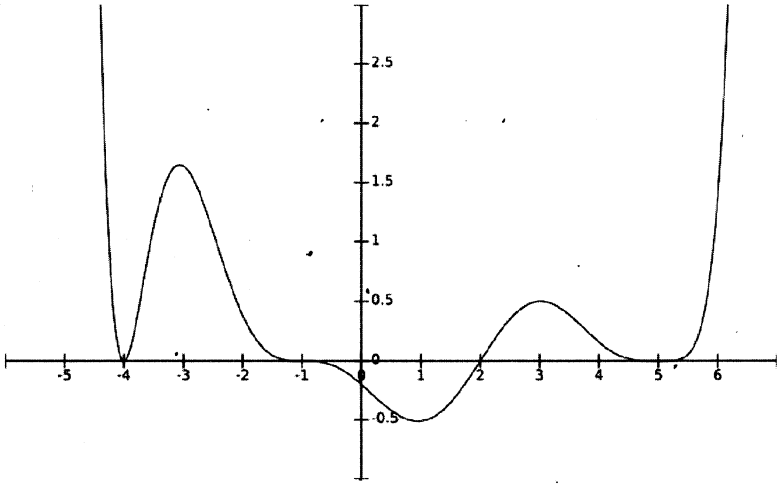
$f(0) = -0(-2)^3(1)^2 = 0$

(0, 0)

(e) Roughly sketch the graph of f. Be sure that your graph correctly illustrates the y-intercept, the end behavior, and the behavior near the x-intercepts.



31. The graph of a polynomial is shown below.



(a) Is the degree even or odd?

← UP ON BOTH ENDS →

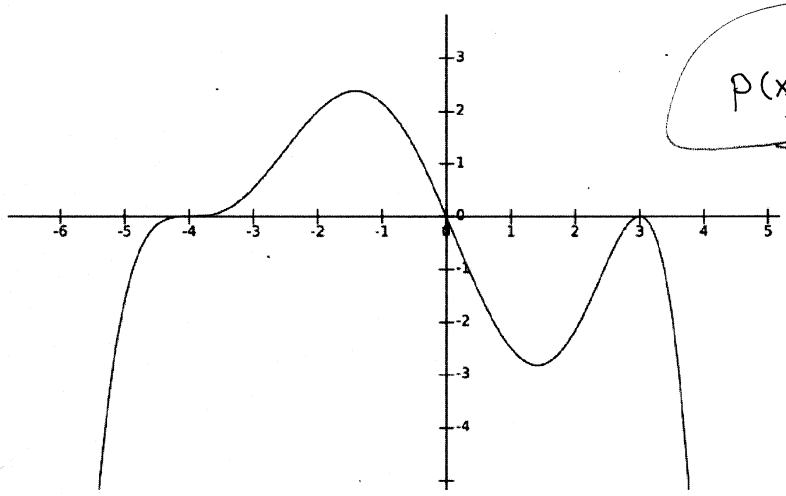
(b) Is the leading coefficient positive or negative?

(c) Which zeros have multiplicity one? $x = 2$

(d) Which zeros have even multiplicity? $x = -4, \; x = 5$

(e) Which zeros have odd multiplicity greater than 1? $x = -1$

32. Give the factored form of a polynomial whose graph has the same general shape of the one given below.



$p(x) = (x+4)^3(x)(x-3)^2$

Objective: Determine the domain of a rational function. [1,13]

33. Determine the domain of the function $f(x) = \dfrac{1}{x^2+5x-6}$.

$(x-1)(x+6)$

ALL REAL #'S EXCEPT $x=1$ & $x=-6$

34. Determine the domain of the function $f(x) = \dfrac{x+4}{2x^2+11x+12}$.

$(2x+3)(x+4)$

ALL REAL #'S EXCEPT $x=-\frac{3}{2}$ & $x=-4$

Objective: Determine the vertical, horizontal, and/or slant asymptotes of the graph of a rational function. [13]

35. Let $f(x) \doteq \dfrac{x^2+1}{(x-3)(x+2)}$. Determine the vertical asymptotes of the graph of f.

$x=3$, $x=-2$ MAKE THE DENOM ZERO BUT NOT THE NUMER.

V.A. $x=3$, $x=-2$

36. Let $g(x) = \dfrac{2x-2}{(x-1)(x-5)}$. Determine the vertical asymptotes of the graph of g.

$\dfrac{2(x-1)}{(x-1)(x-5)}$ AFTER CANCELING,

$x=5$ MAKES DENOM ZERO BUT NOT THE NUMER.

V.A. $x=5$

37. Let $f(x) = \dfrac{x^3+3x^2+1}{x^2-x}$. Determine the slant asymptote and the vertical asymptotes of the graph of f.

$$\begin{array}{r} x+4 \\ x^2-x \overline{)\, x^3+3x^2+0x+1} \\ -(x^3-x^2) \\ \hline 4x^2+0x+1 \\ -(4x^2-4x) \\ \hline 4x+1. \end{array}$$

$f(x) = x+4+\dfrac{4x+1}{x^2-x}$, $x^2-x = x(x-1)$

SLANT ASYMP: $y=x+4$

V.A. $x=0$, $x=1$

38. Determine the horizontal asymptote of the graph of $R(x) = \dfrac{3x^2 + 2x - 9}{2x^2 - x - 1}$.

SAME DEGREE

$y = \frac{3}{2}$

39. Determine the horizontal asymptote of the graph of $H(x) = \dfrac{3x}{6x^2 + 3x + 112}$.

DENOM HAS. GREATER DEG.

$y = 0$

40. Explain how you know that the graph of $f(x) = \dfrac{x^3 - 8}{x + 7}$ has no horizontal asymptote.

Deg of numer = 3 > 1 = Deg of denom

41. Give an example of a rational function whose graph has the horizontal asymptote $y = 3$ and vertical asymptotes $x = 5$ and $x = -9$.

$R(x) = \dfrac{3x^2}{(x-5)(x+9)}$

Objective: Sketch the graph of a rational function. [13]

42. Sketch the graph of $y = \dfrac{2x-4}{x-5}$. $= \dfrac{2(x-2)}{x-5}$

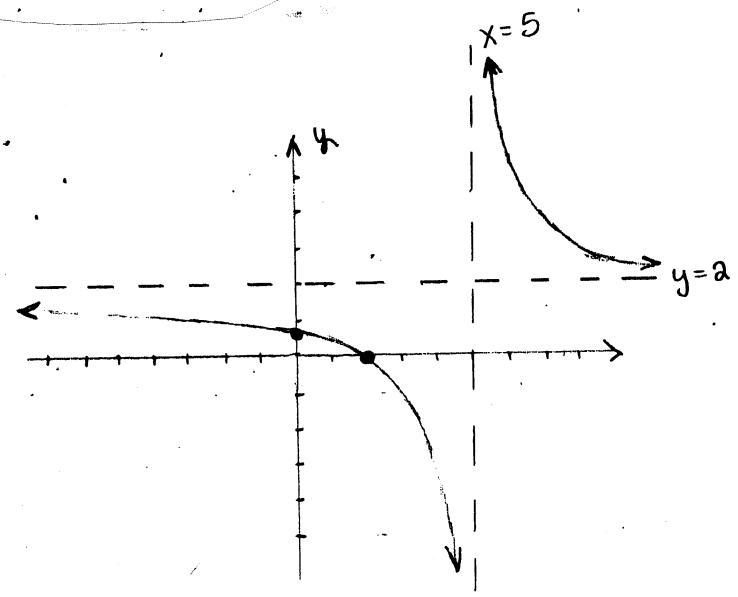
H.A. $y = 2$

V.A. $x = 5$

X-INTERCEPT: $(2, 0)$

y-INTERCEPT: $\left(0, \frac{4}{5}\right)$

PLOT SOME POINTS & USE GRAPHING CALC



43. Sketch the graph of $y = \dfrac{x^2-2x-3}{2x+6}$.

$$y = \frac{(x-3)(x+1)}{2(x+3)} = \frac{1}{2}x - \frac{5}{2} + \frac{12}{2x+6}$$

$$\begin{array}{r} \frac{1}{2}x - \frac{5}{2} \\ 2x+6 \overline{)\, x^2 - 2x - 3} \\ -(x^2+3x) \\ \hline -5x - 3 \\ -(-5x-15) \\ \hline 12 \end{array}$$

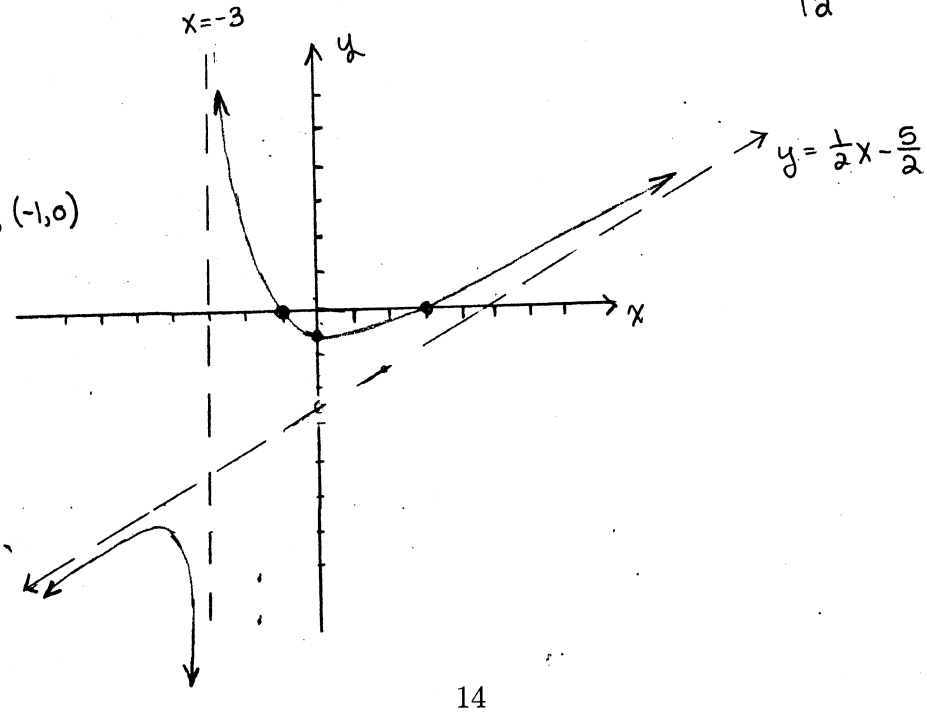
SLANT ASYMP: $y = \frac{1}{2}x - \frac{5}{2}$

V.A. $x = -3$

X-INTS: $(3,0), (-1,0)$

y-INT: $\left(0, -\frac{1}{2}\right)$

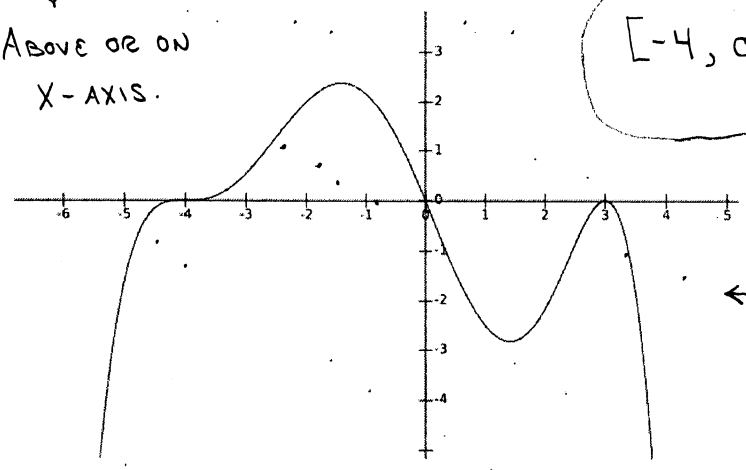
PLOT SOME POINTS & USE GRAPHING CALC.



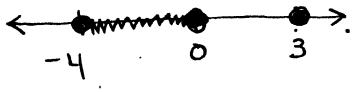
Objective: Solve polynomial inequalities. [11,12]

44. Let $p(x)$ be the polynomial whose graph is shown below. Use the graph to solve the inequality $p(x) \geq 0$.

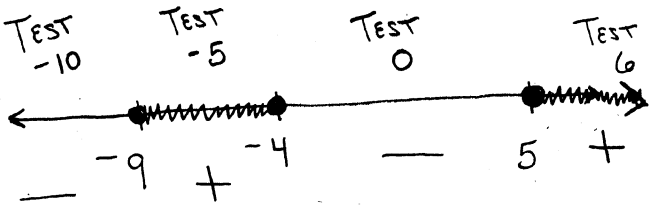
ABOVE OR ON X-AXIS.



$[-4, 0] \cup \{3\}$



45. Solve and graph the solution set on a number line: $(x+4)(x+9)(x-5) \geq 0$



ZEROS ARE $x = -4$, $x = -9$, $x = 5$

$[-9, -4] \cup [5, \infty)$

46. Solve and graph the solution set on a number line: $x^3 + 5x^2 \leq 4x + 20$

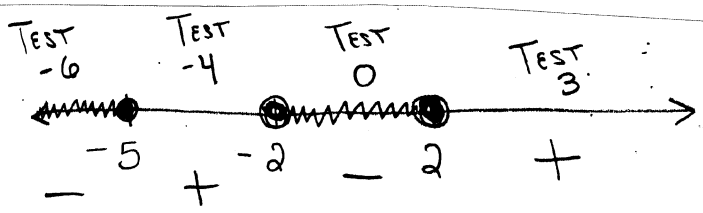
$x^3 + 5x^2 - 4x - 20 \leq 0$

$x^2(x+5) - 4(x+5) \leq 0$

$(x^2 - 4)(x+5) \leq 0$

$(x-2)(x+2)(x+5) \leq 0$

$x = 2 \quad x = -2 \quad x = -5$



$(-\infty, -5] \cup [-2, 2]$

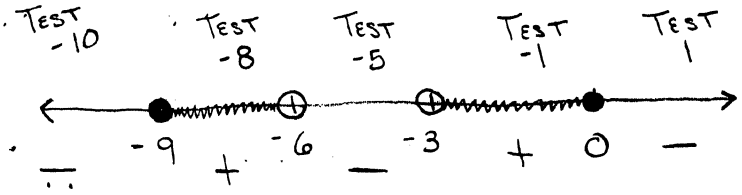
Objective: Solve rational inequalities. [11,12,13]

47. Solve and graph the solution set on a number line: $f(x) = \dfrac{-x^2 - 9x}{x^2 + 9x + 18} \geq 0$

$\dfrac{-x(x+9)}{(x+3)(x+6)} \geq 0$

Zeros: $x = 0,\ x = -9$

Restricted values: $x = -3,\ x = -6$



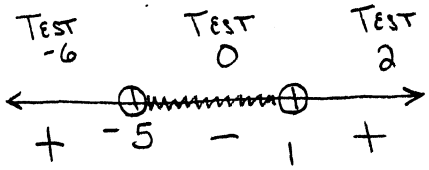
$[-9, 6) \cup (-3, 0]$

48. Solve and graph the solution set on a number line: $f(x) = \dfrac{x-1}{x+5} < 0$

$\dfrac{x-1}{x+5} < 0$

Zeros: $x = 1$

Restrict: $x = -5$



$(-5, 1)$

49. Solve and graph the solution set on a number line: $\dfrac{-1}{x-6} < \dfrac{2}{9-x}$

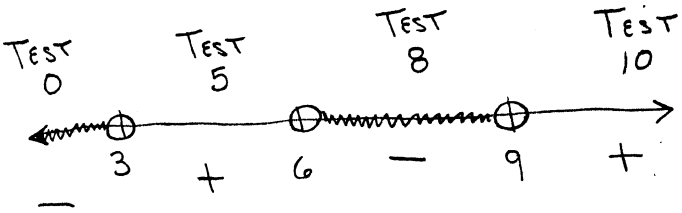
$-\dfrac{1}{x-6} - \dfrac{2}{9-x} < 0$

$\dfrac{-(9-x) - 2(x-6)}{(x-6)(9-x)} < 0$

$\dfrac{-x+3}{(x-6)(9-x)} < 0$

Zeros: $x = 3$

Restrict: $x = 6,\ x = 9$



$(-\infty, 3) \cup (6, 9)$