# Math 129 - Test 3B

November 13, 2019

Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary. Label your axes when graphing. You may get partial credit on multiple choice problems if you show correct work or explanations.

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1. (6 points [5]) The functions $f$ and $g$ are defined in the table shown below. Use the data from the table to evaluate each of the following.

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $f(x)$ | -1 | 3 | 2 | 5 | 0 |
| $g(x)$ | 7 | 0 | 4 | -7 | 4 |

(a) $(g+f)(3)$

$g(3)+f(3) = 4+2 = \boxed{6}$

(b) $(gg)(4)$

$g(4)\,g(4) = (-7)(-7) = \boxed{49}$

(c) $\left(\frac{g}{f}\right)(0)$

$\frac{g(0)}{f(0)}$ IS NOT DEFINED

(d) $(f \circ g)(5)$

$f(g(5)) = f(4) = \boxed{5}$

2. (4 points [5]) Refer to the functions $f$ and $g$ defined in the problem above.

(a) What is the domain of $(g \circ f)$?

$\{2,3,4\}$

(b) What is the range of $(g \circ f)$?

$\{0,4\}$

| x | g(f(x)) |
|---|---|
| 1 | g(-1) DNE |
| 2 | g(3) = 4 |
| 3 | g(2) = 0 |
| 4 | g(5) = 4 |
| 5 | g(0) DNE |

3. (4 points [5]) The formula for the surface area ($S$) of a sphere of radius $r$ is $S = 4\pi r^2$. A spherical balloon is being inflated in such a way that its radius (in inches) after $t$ minutes is given by $r(t) = 3t + 1$. Find a formula for $S$ in terms of the time $t$. Completely expand and simplify your answer.

$$(S \circ r)(t) = 4\pi(3t+1)^2$$
$$= 4\pi(9t^2 + 6t + 1)$$
$$= 36\pi t^2 + 24\pi t + 4\pi$$

4. (6 points [1,5]) Consider the function

$$f(x) = \begin{cases} x^2 + x, & x < -5 \\ |x+6| + 2, & -5 \le x < 1 \\ \sqrt{6x}, & x > 2 \end{cases}$$

(a) Evaluate $f(6)$.

$$f(6) = \sqrt{36} = 6$$

(b) Evaluate $f(-10)$.

$$f(-10) = (-10)^2 + (-10) = 90$$

(c) Evaluate $f(1)$

$f(1)$ is NOT DEFINED

(d) What is the domain of $f$?

$$(-\infty, -5) \cup [-5, 1) \cup (2, \infty)$$
$$= (-\infty, 1) \cup (2, \infty)$$

5. (8 points [1,5,9]) Consider the function

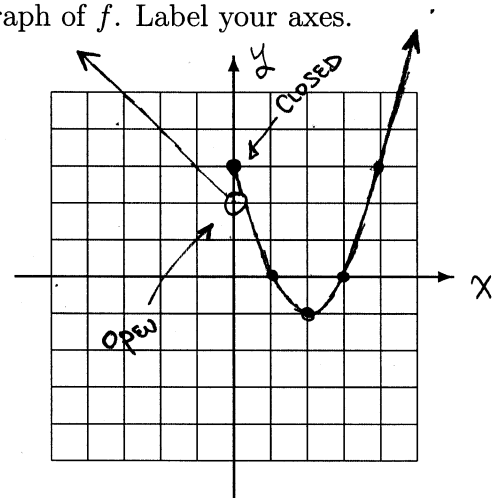
$$f(x) = \begin{cases} -x+2, & x < 0 \\ (x-2)^2 - 1, & x \geq 0 \end{cases}$$

(a) What is the domain of $f$?

$(-\infty, 0) \cup [0, \infty) = (-\infty, \infty) = \mathbb{R}$

(b) Carefully sketch the graph of $f$. Label your axes.

For $x < 0$...
$y = -x + 2$
Line w/
slope $-1$
y-int $(0, 2)$




For $x \geq 0$:...:
$y = x^2$ shifted
right 2
&
down 1

(c) Is $f$ a continuous function? If not, where is it discontinuous?

No, $f$ has a discontinuity at $x = 0$.
(The pieces don't match up.)

6. (4 points [9]) Determine the equations of the horizontal and vertical asymptotes of the graph of $y = \dfrac{1}{x+6} - 8$.

$y = \frac{1}{x}$ shifted left 6, down 8

V.A. $x = -6$

H.A. $y = -8$

7. (3 points) Find two functions $f$ and $g$ so that $(f \circ g)(x) = (7 + 2\sqrt{x})^3$.

$g(x) = 7 + 2\sqrt{x}$

$f(x) = x^3$.

8. (6 points [9]) What are the domain and range of the function $g(x) = 6 + \sqrt{x-3}$? Write your answers in interval notation, and say which answer is which.

$y = \sqrt{x}$

Domain = $[0, \infty)$

Range = $[0, \infty)$

Shift right 3 & up 6 →

Domain = $[3, \infty)$

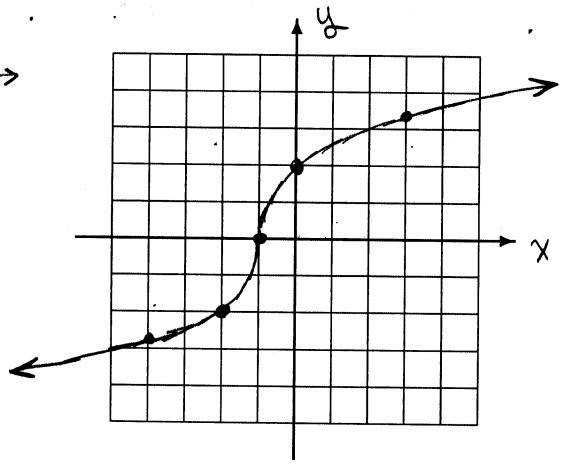
Range = $[6, \infty)$

9. (7 points [9]) Explain how the graph of $g(x) = 2\sqrt[3]{x+1}$ can be obtained from the graph of $f(x) = \sqrt[3]{x}$. Then carefully sketch the graph of $g$. (Label your axes.)

Start with graph of $y = \sqrt[3]{x}$

① Shift left 1 unit

② Vertically stretch by factor of 2 (Double each height.)



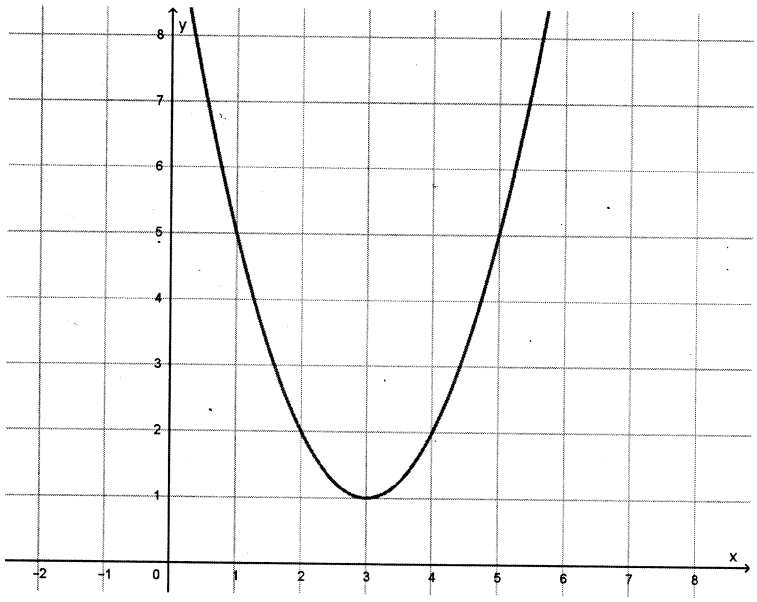

| x | g(x) |
|---|---|
| 0 | 2 |
| -1 | 0 |
| -2 | -2 |
| -4 | $\approx -2.88$ |
| 3 | $\approx 3.17$ |

10. (3 points [9]) The graph of $y = x^2$ is shifted to obtain the new graph shown below. What is an equation for the new graph?

Shift 3 right and 1 up

⇓

$y = (x-3)^2 + 1$

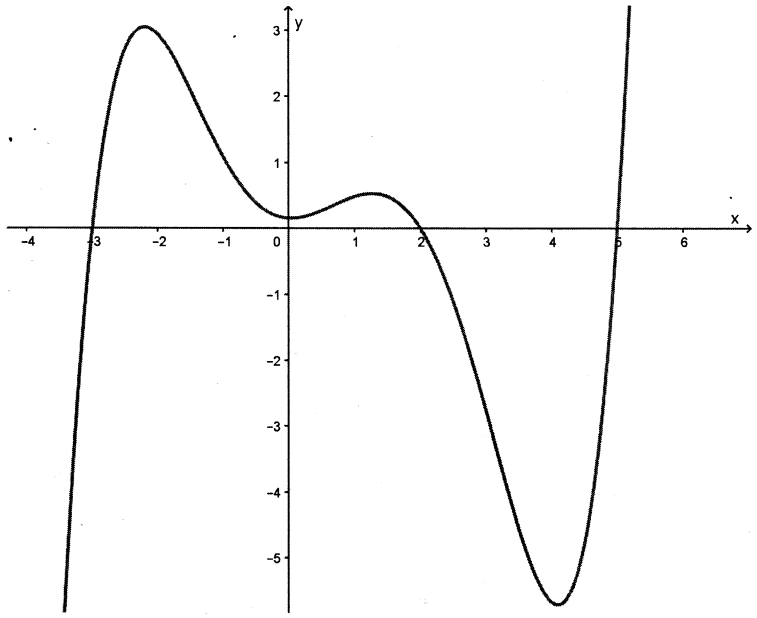


11. (8 points [5]) Evaluate the difference quotient $\frac{g(x+h)-g(x)}{h}$ for the function $g(x) = x^2 + 2x$.
Completely expand and simplify your answer.

$$\frac{[(x+h)^2 + 2(x+h)] - [x^2 + 2x]}{h} = \frac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h}$$

$$= \frac{2xh + h^2 + 2h}{h} = \boxed{2x + h + 2}$$

12. (9 points [8,10]) The graph of the function $f$ is shown below.




(a) Determine the intervals on which $f(x) > 0$.

$(-3, 2) \cup (5, \infty)$

(b) Determine the intervals on which $f$ is decreasing.

About $(-2.25, 0) \cup (1.5, 4.1)$

(c) Determine any relative (local) minimum or maximum values. Say which are which.

Max

$y = 3$ at $x = -2.25$

$y = 0.5$ at $x = 1.5$

Min

$y = -6$ at $x = 4.1$

$y = 0.25$ at $x = 0$

13. (18 points [8,10]) Consider the following types of functions:

**A** – Constant functions, $f(x) = c$

**B** – Linear functions (Not constant), $f(x) = mx + b$, $m \neq 0$

**C** – Absolute value function, $f(x) = |x|$

**D** – Reciprocal function, $f(x) = 1/x$

**E** – Reciprocal square function, $f(x) = 1/x^2$

**F** – Squaring function, $f(x) = x^2$

**G** – Square root function, $f(x) = \sqrt{x}$

**H** – Cubing function, $f(x) = x^3$

**I** – Cube root function, $f(x) = \sqrt[3]{x}$

For each part below, indicate (by letter) which functions have the given property. List all that apply.

(a) The graph has a U shape.

F

(b) The $x$-axis is a horizontal asymptote of the graph.

D, E

(c) The graph is symmetric about the origin.

D, H, I

(d) The graph is a line.

A, B

(e) The domain is the set of all real numbers.

A, B, C, F, H, I

(f) The graph has V shape.

C

(g) The graph is symmetric about the $y$-axis.

A, C, E, F

(h) The graph has a vertical asymptote.

D, E

(i) The range is the set of real numbers.

B, H, I

14. (2 points [8,10]) Which one of the following properties would indicate that the graph of $f$ is symmetric about the origin?

(a) $(f/f)(x) = f(x)$
(b) $f(-x) = -f(x)$
(c) $f(-x) = f(x)$
(d) $f(x) = (f \circ f)(-x)$

15. (2 points [8,10]) Which one of the following properties would indicate that the graph of $f$ is symmetric about the $y$-axis?

(a) $f(-x) = -f(x)$
(b) $f(-x) = -f(-x)$
(c) $f(x) = (f \circ f)(x)$
(d) $f(-x) = f(x)$

16. (2 points [8,10]) Which function has a graph that is symmetric about the $x$-axis?

(a) $f(x) = \dfrac{1}{x-2} + 193$

(b) $g(x) = 63\sqrt[3]{x-4}$
(c) $h(x) = 5x^2 + 55x + 555$
(d) No such function exists. ← EXCEPT THE CONSTANT FUNCTION $f(x) = 0$.

17. (2 points [8,10]) Which function has a graph that is symmetric about the origin?

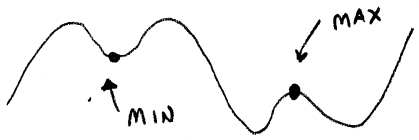
(a) $h(x) = 0.000001x^2$
(b) $g(x) = 0.000001\sqrt{x}$
(c) $f(x) = 0.000001x^3$
(d) No such function exists.

18. (2 points [8,10]) Which function has a graph that is symmetric about the $y$-axis?

(a) $g(x) = 0.0000001x^3$
(b) $f(x) = 8723 - 5135x^2$
(c) $h(x) = \dfrac{829}{x - 517}$
(d) No such function exists.

19. (2 points [8,10]) True or False: A local (relative) maximum value of a function must be greater than a local minimum.

FALSE




MAX < MIN

20. (2 points [8,10]) Suppose you are given the graph of a function $f$, and you would like to solve the inequality $f(x) > 0$. Which of the following should you do?

(a) Identify the portions of the $x$-axis where the graph is lying below the axis.

(b) Identify the portions of the $x$-axis where the graph is lying above the axis.

(c) Identify the portions of the $x$-axis where the graph is falling to the right.

(d) Identify the portions of the $x$-axis where the graph is rising to the right.