Math 129 - Final Exam A

December 11, 2019

Name key

Score

Show all work to receive full credit. Supply explanations where necessary. Label your axes when graphing.

1. (4 points [11]) Solve for r: $-2|3r-7| = -12$

$|3r-7| = 6$

$3r - 7 = \pm 6$

$3r = 13$

OR

$3r = 1$

$$\boxed{r = \frac{13}{3} \qquad r = \frac{1}{3}}$$

2. (6 points [3]) Solve for y. Write your solution set in interval notation, and graph it on a number line.

$$2(y+2) - 3 < y + 7 \quad \text{and} \quad 7 - 2y \leq 1$$

$2y + 4 - 3 < y + 7$

$2y + 1 < y + 7$

$y < 6$

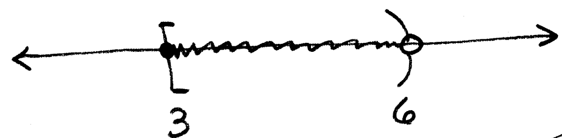
-AND-

$7 - 2y \leq 1$

$-2y \leq -6$

$y \geq 3$

$[3, 6)$



3. (5 points [7]) Solve for x. Write your answer(s) in decimal form, rounded to the nearest hundredth.

$$4x^2 - 4x - 1 = 0$$

Quad. Formula.

$a = 4$

$b = -4$

$c = -1$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(4)(-1)}}{2(4)}$$

$$= \frac{4 \pm \sqrt{32}}{8} = \frac{4 \pm 4\sqrt{2}}{8} = \frac{1}{2} \pm \frac{\sqrt{2}}{2}$$

$$\boxed{x \approx 1.21 \quad \text{OR} \quad x \approx -0.21}$$

4. (4 points [3,11]) Solve for x: $\dfrac{3}{x} = \dfrac{4}{2x+1}$

$3(2x+1) = 4x$

$6x + 3 = 4x$

$2x = -3$

$\boxed{x = -\frac{3}{2}}$

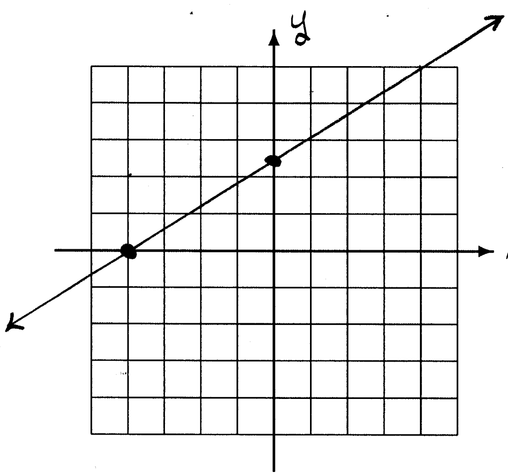
5. (6 points [3]) A line is described by the equation $-\frac{5}{4}x + 2y = 5$. Find the x- and y-intercepts of the line. Then plot your intercepts and sketch the line.

X- INT...

$y = 0 \Rightarrow -\frac{5}{4}x = 5$

$x = -4$

$\boxed{\text{X-INT: } (-4,0)}$



Y- INT...

$x = 0 \Rightarrow 2y = 5$

$y = \frac{5}{2}$

$\boxed{\text{Y-INT: } (0, \tfrac{5}{2})}$

6. (5 points [2,4]) The line L passes through the point $(-4,-2)$ and is perpendicular to the line given by $y = -2x + 1$. Find an equation for the line L. Write your final answer in standard form ($Ax + By = C$).

SLOPE OF L MUST BE $\frac{1}{2}$
(OPP. RECIP OF -2)

POINT $(-4,-2)$

$y + 2 = \frac{1}{2}(x+4)$

$y + 2 = \frac{1}{2}x + 2$

$y = \frac{1}{2}x$

$\boxed{\frac{1}{2}x - y = 0}$

7. (3 points [1]) Determine the domain of the function $f(x) = \dfrac{2x}{(2x+1)(x-7)}$.

$2x + 1 = 0$

$\Rightarrow x = -\frac{1}{2}$

$x - 7 = 0$

$\Rightarrow x = 7$

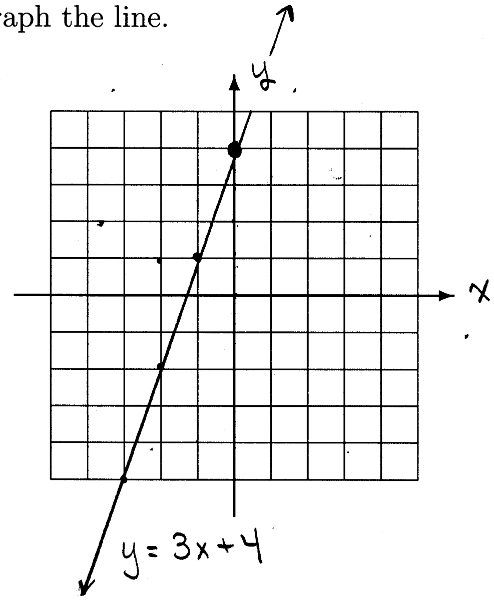
DOMAIN OF f = ALL REAL #'S EXCEPT $x = -\frac{1}{2}$ & $x = 7$

8. (4 points [2,4]) Determine the slope and y-intercept of the line described by $\underbrace{3x - y = -4}$. Then graph the line.

$3x + 4 = y$

Slope $= 3$

Y-INT is $(0,4)$



9. (4 points [2,3,4]) A street vendor will sell 200 ice cream cones if she sells them for \$2 each, and she will sell 120 cones if she sells them for \$3 each. Determine the linear equation that describes how the demand varies with cost. Say what your variables represent.

$\boxed{\begin{array}{l} X = \text{COST} \\ y = \text{DEMAND} \end{array}}$

POINTS...

$(2, 200)$

$(3, 120)$

$m = \dfrac{120 - 200}{3 - 2} = -80$

$y = -80x + 360$

$y = -80x + b$

$200 = -80(2) + b \Rightarrow b = 360$

10. (6 points [5]) Let $g(x) = x^2 + 2x$. Expand and simplify the difference quotient $\dfrac{g(x+h) - g(x)}{h}$.

$$\frac{g(x+h) - g(x)}{h} = \frac{[(x+h)^2 + 2(x+h)] - [x^2 + 2x]}{h}$$

$$= \frac{(x^2 + 2xh + h^2 + 2x + 2h) - (x^2 + 2x)}{h} = \frac{\cancel{x^2} + 2xh + h^2 + \cancel{2x} + 2h - \cancel{x^2} - \cancel{2x}}{h}$$

$$= \frac{2xh + h^2 + 2h}{h} = \frac{\cancel{h}(2x + h + 2)}{\cancel{h}} = 2x + h + 2$$

11. (7 points [8,9,10]) Let $f(x) = 2|x+1| - 3$.

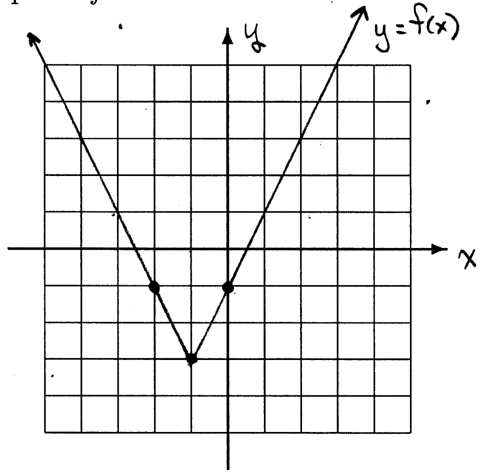
(a) Explain how the graph of f can be obtained from the graph of $y = |x|$.

① SHIFT LEFT 1 UNIT

② STRETCH BY FACTOR OF 2 (DOUBLING EACH y COORD)

③ SHIFT DOWN 3 UNITS

(b) Carefully sketch the graph of f.



(c) Determine the domain and range of f.

D: ALL REAL #'S

R: $[-3, \infty)$

12. (6 points [5]) Let $f(x) = 3 + \sqrt{x}$ and $g(x) = \begin{cases} x^2+4, & \text{if } x < -2 \\ 3x+7, & \text{if } x > 0 \end{cases}$.

(a) Compute $g(-1)$.

$\boxed{\text{NOT DEFINED}}$

(b) Compute $g(-10)$.

$(-10)^2 + 4 = \boxed{104}$

(c) Compute $(g \circ f)(4)$.

$g(f(4)) = g(5) = 3(5)+7 = \boxed{22}$

(d) If $h(x) = x^2+1$, then what function is $(h \circ f)(x)$? Completely expand and simplify your answer.

$h(f(x)) = (3+\sqrt{x})^2 + 1 = 9 + 6\sqrt{x} + x + 1$

$= \boxed{10 + 6\sqrt{x} + x}$

13. (12 points [11,12,13]) Consider the polynomial $f(x) = -2x(x-2)^3(x+1)^2$.

(a) Determine the degree of f.

$1 + 3 + 2 = 6$

(b) State the zeros of f and their corresponding multiplicities.

$X = 0$, mult 1 $\quad$ $X = -1$, mult 2

$X = 2$, mult 3

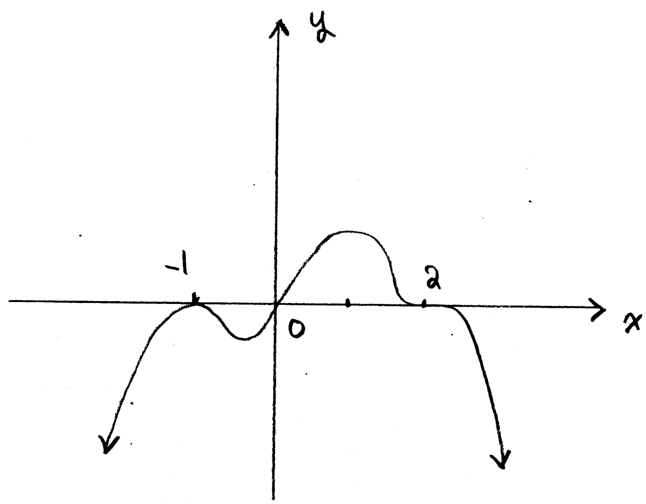
(c) Describe the end behavior of the graph of f.

Neg leading coeff & deg 6 $\Rightarrow$ Down left / Down right

(d) Determine the y-intercept.

$X = 0 \Rightarrow y = f(0) = 0$ $\quad$ $(0,0)$

(e) Roughly sketch the graph of f. Be sure that your graph correctly illustrates the y-intercept, the end behavior, and the behavior at the x-intercepts.



(f) Use your graph to solve $f(x) > 0$. Write your solution in interval notation.

$f(x) > 0$ on $(0, 2)$

14. (4 points [8]) The graph of $f(x) = (x+3)^2 - 5$ is a parabola.

(a) Explain how the graph of f can be obtained from the graph of $y = x^2$.

SHIFT 3 UNITS LEFT AND 5 UNITS DOWN.

(b) Determine the vertex and an equation for the axis of symmetry of the graph of f.

VERTEX: $(-3, -5)$ SYMMETRY AXIS: $X = -3$

15. (8 points [13]) Let $f(x) = \dfrac{x^3 + 5x^2 + 7}{x^2 - 2x}$. Determine the slant asymptote and the vertical asymptotes of the graph of f.

$$
\begin{array}{r}
X + 7 \\
x^2 - 2x \,\overline{)\, X^3 + 5x^2 + 0x + 7} \\
\underline{-(X^3 - 2x^2)} \\
7x^2 + 0x + 7 \\
\underline{-(7x^2 - 14x)} \\
14x + 7
\end{array}
$$

$$f(x) = x + 7 + \frac{14x + 7}{x(x-2)}$$

SLANT ASYMP: $y = x + 7$

ZEROS OF DENOM, NOT NUMER...

V.A. $x = 0$, $x = 2$

16. (4 points [13]) Let $R(x) = \dfrac{2x^3 + x^2}{x(x-3)(x+7)}$.

(a) Determine any horizontal asymptotes of the graph of R.

LEADING TERM OF NUMER $= 2x^3$

LEADING TERM OF DENOM $= x^3$

H.A. $y = 2$

(b) Explain why $x = 0$ is NOT a vertical asymptote of the graph of R.

$$\frac{2x^3 + x^2}{x(x-3)(x+7)} = \frac{2x^2 + x}{(x-3)(x+7)}$$

THE FACTOR OF X IN THE DENOM CANCELS.

17. (4 points [12]) Use synthetic division and the remainder theorem to evaluate $f(2)$ if $f(x) = 2x^2 + 3x + 1$.

$$
\begin{array}{r|rrr}
2 & 2 & 3 & 1 \\
+ & & 4 & 14 \\
\hline
 & 2 & 7 & 15
\end{array}
\longrightarrow \boxed{f(2) = 15}
$$

18. (8 points [11,12,13]) Solve the inequality and write your solution in interval notation. Show all work.

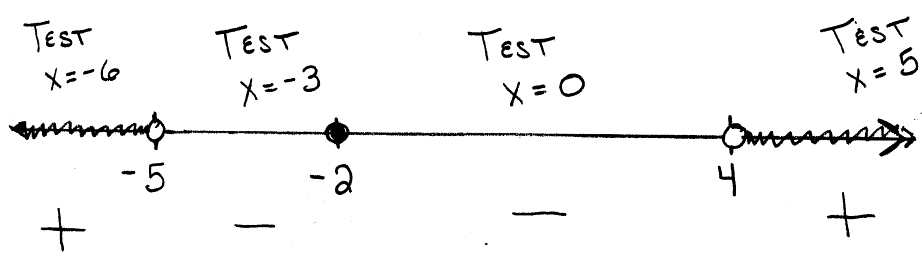
$$\frac{(x+2)^2}{(x-4)(x+5)} \geq 0$$

ZEROS OF NUMER:

$X = -2$

ZEROS OF DENOM:

$X = 4, \; X = -5$



$f(x) \geq 0$ ON

$(-\infty, -5) \cup (4, \infty) \cup \{-2\}$