Math 129 - Final Exam B

December 11, 2019

Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary. Label your axes when graphing.

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1. (4 points [11]) Solve for r: $-3|7-2r| = -12$

$|7-2r| = 4$

$7-2r = \pm 4$

$-2r = -3$ or $-2r = -11$

$r = \frac{3}{2}$ or $r = \frac{11}{2}$

2. (6 points [3]) Solve for w. Write your solution set in interval notation, and graph it on a number line.

$$3(w+1)-5 < w+8 \quad \text{and} \quad 10-3w \leq 1$$

$3w+3-5 < w+8$

$3w-2 < w+8$

$2w < 10$

$w < 5$

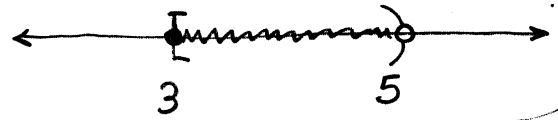
-AND-

$10-3w \leq 1$

$-3w \leq -9$

$w \geq 3$

$[3, 5)$



3. (5 points [7]) Solve for x. Write your answer(s) in decimal form, rounded to the nearest hundredth.

$$2x^2 - 3x - 1 = 0$$

QUAD. FORMULA

$a = 2$

$b = -3$

$c = -1$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{3 \pm \sqrt{17}}{4}$$

$x = \frac{3+\sqrt{17}}{4} \approx 1.78$

$x = \frac{3-\sqrt{17}}{4} \approx -0.28$

4. (4 points [3,11]) Solve for x: $\dfrac{5}{x} = \dfrac{8}{2x-1}$

$5(2x-1) = 8x$

$10x - 5 = 8x$

$2x = 5$ → $x = \frac{5}{2}$

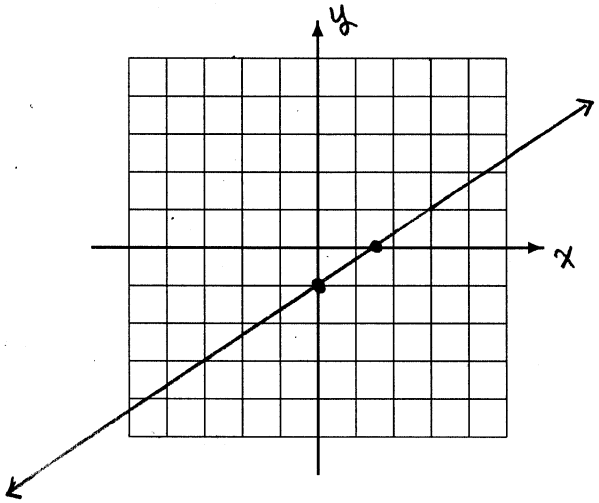
5. (6 points [3]) A line is described by the equation $\frac{8}{3}x - 4y = 4$. Find the x- and y-intercepts of the line. Then plot your intercepts and sketch the line.

X-INT...

$y = 0 \Rightarrow \frac{8}{3}x = 4$

$x = \frac{12}{8} = \frac{3}{2}$

X-INT: $(\frac{3}{2}, 0)$



Y-INT ...

$x = 0 \Rightarrow -4y = 4$

$y = -1$

Y-INT: $(0, -1)$

6. (5 points [2,4]) The line L passes through the point $(6, 4)$ and is perpendicular to the line given by $y = -2x + 1$. Find an equation for the line L. Write your final answer in standard form $(Ax + By = C)$.

SLOPE OF L MUST BE $\frac{1}{2}$ (OPP. RECIP OF -2)

POINT (6,4)

$y - 4 = \frac{1}{2}(x - 6)$

$y - 4 = \frac{1}{2}x - 3$

$\frac{1}{2}x - y = -1$

7. (3 points [1]) Determine the domain of the function $f(x) = \dfrac{2(x-1)}{(2x+5)(x-3)}$.

$2x + 5 = 0 \Rightarrow x = -\frac{5}{2}$ $\qquad$ $x - 3 = 0 \Rightarrow x = 3$

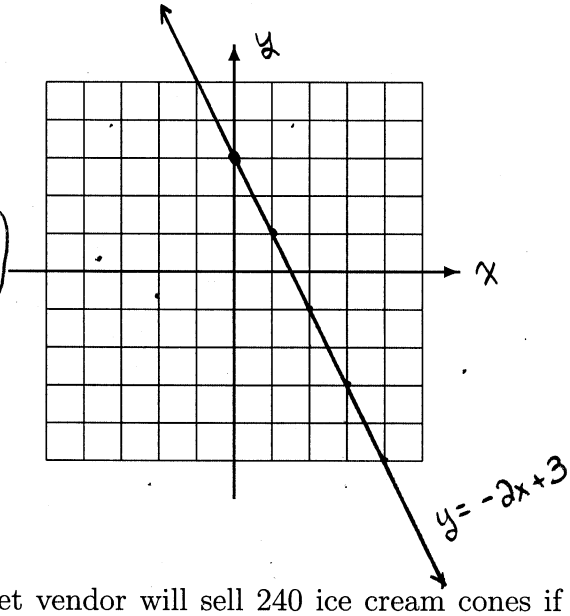
DOMAIN OF f = ALL REAL #'S EXCEPT $x = -\frac{5}{2}$ & $x = 3$

8. (4 points [2,4]) Determine the slope and y-intercept of the line described by $2x + y = 3$. Then graph the line.

$y = -2x + 3$

Slope $= -2$

Y-INT IS $(0,3)$



RISE $= -2$

RUN $= 1$

9. (4 points [2,3,4]) A street vendor will sell 240 ice cream cones if she sells them for \$2 each, and she will sell 150 cones if she sells them for \$3 each. Determine the linear equation that describes how the demand varies with cost. Say what your variables represent.

x = COST

y = DEMAND

POINTS...

$(2, 240)$

$(3, 150)$

$$m = \frac{150 - 240}{3 - 2} = -90$$

$y = -90x + b$

$240 = -90(2) + b \Rightarrow b = 420$

$y = -90x + 420$

10. (6 points [5]) Let $g(x) = x^2 + 5x$. Expand and simplify the difference quotient $\dfrac{g(x+h) - g(x)}{h}$.

$$\frac{g(x+h) - g(x)}{h} = \frac{\left[(x+h)^2 + 5(x+h)\right] - \left[x^2 + 5x\right]}{h}$$

$$= \frac{(x^2 + 2xh + h^2 + 5x + 5h) - (x^2 + 5x)}{h} = \frac{x^2 + 2xh + h^2 + 5x + 5h - x^2 - 5x}{h}$$

$$= \frac{2xh + h^2 + 5h}{h} = \frac{h(2x + h + 5)}{h} = 2x + h + 5$$

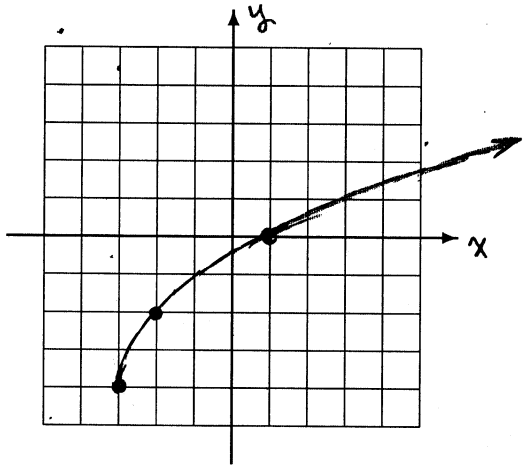
11. (7 points [8,9,10]) Let $f(x) = 2\sqrt{x+3} - 4$.

(a) Explain how the graph of f can be obtained from the graph of $y = \sqrt{x}$

① SHIFT LEFT 3 UNITS ③ SHIFT DOWN 4 UNITS

② STRETCH VERTICALLY BY FACTOR OF 2 (DOUBLING EACH y-VALUE)

(b) Carefully sketch the graph of f.



(c) Determine the domain and range of f.

DOMAIN = $[-3, \infty)$

RANGE = $[-4, \infty)$

12. (6 points [5]) Let $f(x) = 3 + \sqrt{x}$ and $g(x) = \begin{cases} x^2 + 4, & \text{if } x < -2 \\ 3x + 7, & \text{if } x > 0 \end{cases}$

(a) Compute $g(-10)$.

$(-10)^2 + 4 =$ **104**

(b) Compute $g(-1)$.

NOT DEFINED

(c) Compute $(g \circ f)(9)$.

$g(f(9)) = g(6) = 3(6) + 7 =$ **25**

(d) If $h(x) = x^2 + 3$, then what function is $(h \circ f)(x)$? Completely expand and simplify your answer.

$h(f(x)) = (3 + \sqrt{x})^2 + 3 = 9 + 6\sqrt{x} + x + 3$

$= 12 + 6\sqrt{x} + x$

13. (12 points [11,12,13]) Consider the polynomial $f(x) = 3x(x-3)^2(x+2)^3$.

(a) Determine the degree of f.

$1+2+3 = \boxed{6}$

(b) State the zeros of f and their corresponding multiplicities.

$x = 0$, mult 1

$x = 3$, mult 2

$x = -2$, mult 3

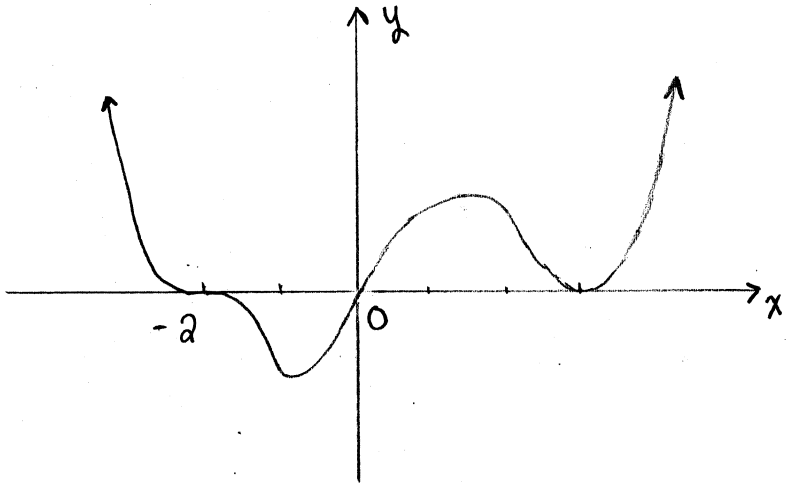
(c) Describe the end behavior of the graph of f.

Pos. leading coeff. and deg 6 $\Rightarrow$ Up left / Up right

(d) Determine the y-intercept.

$x = 0 \Rightarrow y = f(0) = 0$ $\quad (0,0)$

(e) Roughly sketch the graph of f. Be sure that your graph correctly illustrates the y-intercept, the end behavior, and the behavior at the x-intercepts.



(f) Use your graph to solve $f(x) > 0$. Write your solution in interval notation.

$f(x) > 0$ on $(-\infty,-2) \cup (0,3) \cup (3,\infty)$

14. (4 points [8]) The graph of $f(x) = (x-4)^2 + 3$ is a parabola.

(a) Explain how the graph of f can be obtained from the graph of $y = x^2$.

SHIFT 4 UNITS RIGHT AND 3 UNITS UP

(b) Determine the vertex and an equation for the axis of symmetry of the graph of f.

VERTEX: $(4,3)$ SYMMETRY AXIS: $x = 4$

15. (8 points [13]) Let $f(x) = \dfrac{x^3 - 4x^2 + 8}{x^2 + 2x}$. Determine the slant asymptote and the vertical asymptotes of the graph of f.

$$x^2+2x \overline{)\, x^3 - 4x^2 + 0x + 8} \quad x - 6$$

$$-(x^3 + 2x^2)$$

$$-6x^2 + 0x + 8$$

$$-(-6x^2 - 12x)$$

$$12x + 8$$

$$f(x) = x - 6 + \frac{12x+8}{x(x+2)}$$

SLANT ASYMP: $y = x + 3$

ZEROS OF DENOM, NOT NUMER ...

V.A. $x = 0$, $x = -2$

16. (4 points [13]) Let $R(x) = \dfrac{x^3 + x^2}{7x(x+3)(x-9)}$.

(a) Determine any horizontal asymptotes of the graph of R.

LEADING TERM OF NUMER $= x^3$

LEADING TERM OF DENOM $= 7x^3$ $\Longrightarrow$ H.A. $y = \frac{1}{7}$

(b) Explain why $x = 0$ is NOT a vertical asymptote of the graph of R.

$$\frac{x^3 + x^2}{7x(x+3)(x-9)} = \frac{x^2 + x}{7(x+3)(x-9)}$$

THE FACTOR OF x IN THE DENOM CANCELS.

17. (4 points [12]) Use synthetic division and the remainder theorem to evaluate $f(3)$ if $f(x) = 2x^2 - 4x + 5$.

$$\begin{array}{r|rrr} 3 & 2 & -4 & 5 \\ & + & 6 & 6 \\ \hline & 2 & 2 & 11 \end{array}$$

$f(3) = 11$

18. (8 points [11,12,13]) Solve the inequality and write your solution in interval notation. Show all work.

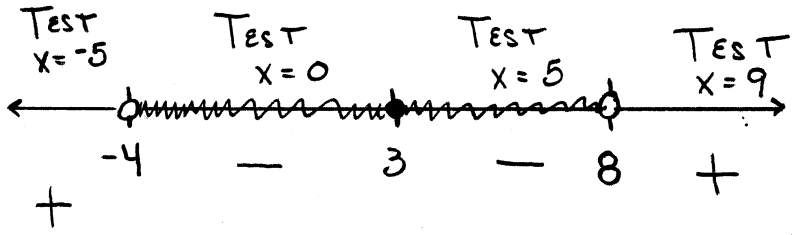
$$\frac{(x-3)^2}{(x+4)(x-8)} \leq 0$$

ZEROS OF NUMER:

$x = 3$

ZEROS OF DENOM:

$x = -4, \; x = 8$



$f(x) \leq 0$

ON

$(-4, 8)$