Math 129 - Review 2

[redacted]

Name key

These problems may help you review for Test 2. They are coded to match the course objectives from your syllabus. Your actual test will not be as long as this review packet. Unless otherwise indicated, you should simplify all answers by reducing fractions, simplifying radicals, and/or rationalizing denominators (as you've done on your ALEKS homework).

Objective: Solve radical equations. [11]

1. Solve for x: $\sqrt[3]{2x-5}+4=2$

$$\left(\sqrt[3]{2x-5}\right)^3=(-2)^3$$
$$2x-5=-8$$
$$2x=-3$$
$$\boxed{x=-\frac{3}{2}}$$

2. Solve for w: $\sqrt[4]{7w+19}+10=2$

$$\sqrt[4]{7w+19}=-8$$

No real solution.

Even-indexed radicals can't be negative.

3. Solve for t: $\sqrt{8t+4}-2=t$

$$\left(\sqrt{8t+4}\right)^2=(t+2)^2$$
$$8t+4=t^2+4t+4$$
$$t^2-4t=0$$
$$t(t-4)=0$$
$$\boxed{t=0 \text{ or } t=4}$$

4. Solve for x: $\sqrt{3x+4}-\sqrt{7x}=-2$

$$\left(\sqrt{3x+4}\right)^2=\left(\sqrt{7x}-2\right)^2$$
$$3x+4=7x-4\sqrt{7x}+4$$
$$4\sqrt{7x}=4x$$
$$\sqrt{7x}=x$$
$$7x=x^2$$
$$x^2-7x=0$$
$$x(x-7)=0$$
$$\cancel{x=0} \text{ or } \boxed{x=7}$$

Objective: Solve equations involving rational exponents. [11]

5. Solve for r: $3(r+1)^{3/4} - 9 = 15$

$3(r+1)^{3/4} = 24$

$(r+1)^{3/4} = 8$

$r+1 = 8^{4/3} = 16$

$\boxed{r = 15}$

6. Solve for x: $(2x-3)^{2/3} = 4$

$2x-3 = 4^{3/2} = 8$

$2x = 11$

$\boxed{x = \frac{11}{2}}$

7. Find the x-intercepts of the graph of $f(x) = -3(x-2)^{4/5} + 48$.

$-3(x-2)^{4/5} + 48 = 0$

$(x-2)^{4/5} = 16$

$x-2 = 16^{5/4} = 32$

$\boxed{x = 34}$

Objective: Solve equations that are quadratic in form. [7,11]

8. Solve by using a substitution: $x^{2/3} - 2x^{1/3} - 15 = 0$

$u = x^{1/3}$

$u^2 - 2u - 15 = 0$

$(u-5)(u+3) = 0$

$u = 5, \ u = -3$

$\boxed{x = 125, \ x = -27}$

9. Solve by using a substitution: $(y^2 - y) - 8(y^2 - y) + 12 = 0$

$u = y^2 - y$

$u^2 - 8u + 12 = 0$

$(u-2)(u-6) = 0$

$u = 2, \ u = 6$

$y^2 - y - 2 = 0 \qquad y^2 - y^2 - 6 = 0$

$(y-2)(y+1) = 0 \qquad (y-3)(y+2) = 0$

$\boxed{y = 2, y = -1 \quad y = 3, y = -2}$

10. Solve by using a substitution: $x^{-2} - 2x^{-1} - 35 = 0$

$u = x^{-1}$

$u^2 - 2u - 35 = 0$

$(u-7)(u+5) = 0$

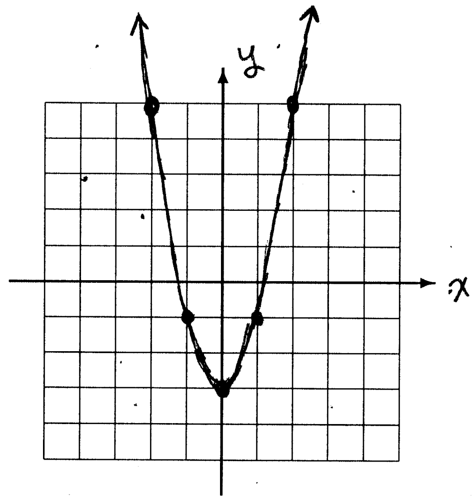
$u = 7 \quad u = -5$

$\boxed{x = \frac{1}{7}, \ x = -\frac{1}{5}}$

Objective: Graph two-variable equations in the rectangular coordinate system. [1,8,9,10]

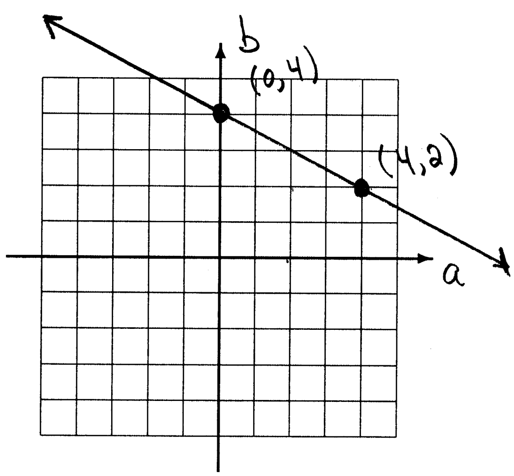
11. Determine five points on the graph of $y = 2x^2 - 3$. Then plot your points and sketch the graph.

x	y
0	-3
1	-1
-1	-1
2	5
-2	5



12. Determine two points on the graph of $b = -\frac{1}{2}a + 4$. Then sketch the graph. Be sure to label your axes.

a	b
0	4
4	2



13. Find a solution of the equation $z^2 + 2a = 4$. Write your solution as an ordered pair.

$$a = 2 \Rightarrow z^2 + 4 = 4$$
$$\Rightarrow z^2 = 0$$
$$\Rightarrow z = 0$$
$$(a, z) = (2, 0)$$

Objective: Find the distance between two points in the rectangular coordinate system.

14. Find the distance from $(-1, 4)$ to $(5, -3)$.

$$\sqrt{(5+1)^2+(-3-4)^2} = \sqrt{36+49} = \sqrt{85}$$

15. Find the distance from the point on the graph of $y = x^2 + 1$ where $x = -1$ to the point on the graph where $x = 3$.

$x=-1$ $\quad$ $x=3$

$y=2$ $\quad$ $y=10$

$$\sqrt{(3+1)^2+(10-2)^2} = \sqrt{16+64} = \sqrt{80} = 4\sqrt{5}$$

16. A diameter of a circle has endpoints $(1, -4)$ and $(9, 0)$. Find the radius of the circle.

$$d = \sqrt{(9-1)^2+(4)^2} = \sqrt{64+16} = \sqrt{80} = 4\sqrt{5}$$

$$r = 2\sqrt{5}$$

Objective: Determine the midpoint of two points.

17. Find the midpoint of the line segment connecting $(1, -5)$ and $(4, -8)$.

$$\left(\frac{1+4}{2}, \frac{-5+(-8)}{2}\right) = \left(\frac{5}{2}, \frac{-13}{2}\right)$$

18. A diameter of a circle has endpoints $(1, -4)$ and $(9, 0)$. Find the center of the circle.

$$\left(\frac{1+9}{2}, \frac{-4+0}{2}\right) = (5, -2)$$

Objective: Use the standard form equation of a circle. [9,10]

19. Determine the center and radius of the circle described by $(x+7)^2 + (y-2)^2 = 12$.

CENTER ... $(h,k) = (-7,2)$

RADIUS ... $r = \sqrt{12} = 2\sqrt{3}$

20. Determine the center and radius of the circle described by $x^2 + y^2 + 2x - 8y + 8 = 0$.

$x^2 + 2x \quad + y^2 - 8y \quad = -8$

$+1 \qquad +16 \qquad + 1 + 16$

$(x+1)^2 + (y-4)^2 = 9$

CENTER ... $(h,k) = (-1,4)$

RADIUS ... $r = 3$

21. A circle centered at $(-1,-3)$ has a diameter of length 8. Find the standard form equation for the circle.

$r = 4$

$(x+1)^2 + (y+3)^2 = 16$

22. A diameter of a circle has endpoints $(1,-4)$ and $(9,0)$. Find the standard form equation for the circle.

SEE PREVIOUS PROBLEMS...

$(h,k) = (5,-2)$

$r = 2\sqrt{5}$

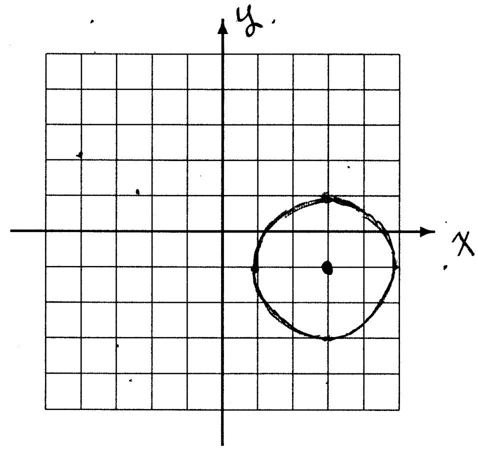
$(x-5)^2 + (y+2)^2 = 20$

Objective: Graph circles. [9,10]

23. Sketch the graph of the equation $(x-3)^2+(y+1)^2=4$.

$(h,k) = (3,-1)$

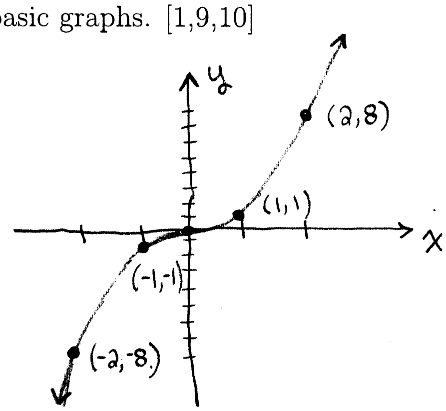
$r = 2$



Objective: Gain familiarity with some basic graphs. [1,9,10]

24. Sketch a detailed graph of $y=x^3$.

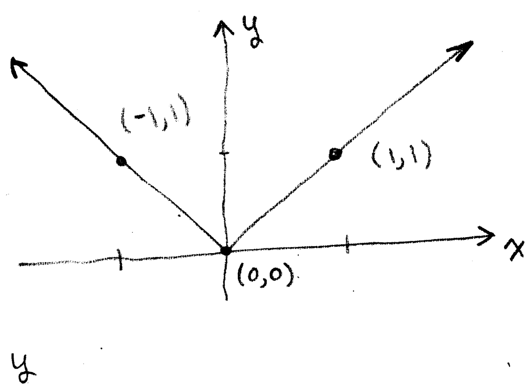
x	y
0	0
±1	±1
±2	±8



25. Sketch a detailed graph of $y=|x|$.

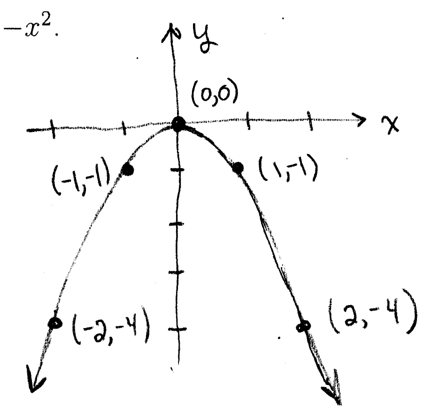
$y = |x| \rightarrow$ $y = x,\ x \geq 0$

$y = -x,\ x < 0$



26. Sketch a detailed graph of $y=-x^2$.

x	y
0	0
1	-1
-1	-1
2	-4
-2	-4



Objective: Determine solutions of two-variable linear equations. [3]

27. Find two points on the line described by $y = 5x - 2$.

$x = 0$

$y = -2$

$\boxed{(0, -2)}$

$x = 1$

$y = 5 - 2 = 3$

$\boxed{(1, 3)}$

28. Find three points on the line described by $3x + 5y = 16$.

$(\frac{1}{3}, 3)$ $\quad 3(\frac{1}{3}) + 5(3) = 1 + 15 = 16$

$\boxed{(2, 2)}$ $\quad 3(2) + 5(2) = 6 + 10 = 16$

$(5, \frac{1}{5})$

$3(5) + 5(\frac{1}{5}) =$
$15 + 1 = 16$

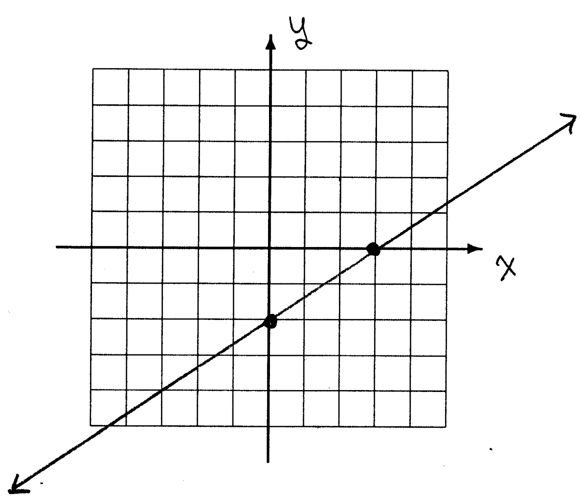
Objective: Graph a line by finding two points on the line. [3]

29. Find two points on the line described by $2x - 3y = 6$. Then plot the points and sketch the graph of the line.

x	y
0	-2
3	0

$(0, -2)$

$(3, 0)$



Objective: Find the x- and y-intercepts of a line. [3]

30. Find the x- and y-intercepts of the line described by $-5x + 8y = 16$. Write your intercepts as ordered pairs, and say which is which.

$x = 0 \Rightarrow 8y = 16$

$y = 2$

(0, 2)

Y-INTERCEPT

$y = 0 \Rightarrow -5x = 16$

$x = \frac{-16}{5} = -3.2$

$(-\frac{16}{5}, 0)$

X-INTERCEPT

Objective: Compute the slope of a line and interpret it as a rate of change. [2,4]

31. A line passes through the two points $(8, 3)$ and $(-2, 4)$. Compute the slope of the line. From one point on the line to another, the y-coordinates by -8. What is the corresponding change in x-coordinates?

↑ CHANGE

$$m = \frac{\Delta y}{\Delta x} = \frac{1}{-10} = \boxed{-\frac{1}{10}.}$$

$$m = \frac{-1}{10} = \frac{-8}{\Delta x} \Rightarrow \boxed{\Delta x = 80}$$

32. A line has slope $-3/4$. In moving from one point on the line to another, the change in x-coordinates was 6 units. What was the corresponding change in y-coordinates?

$$-\frac{3}{4} = \frac{\Delta y}{6} \Rightarrow \Delta y = \frac{-18}{4} = \boxed{-\frac{9}{2}}$$

Objective: Identify equations of horizontal or vertical lines and graph them. [2,3,4]

33. Determine an equation of the line passing through the two points $\underbrace{(-3, 6) \text{ and } (-3, -10)}$.

VERTICAL

$$\boxed{x = -3}$$

34. Determine an equation of the horizontal line through $(9, 13)$.

$$\boxed{y = 13}$$

Objective: Determine lines parallel or perpendicular to given lines. [2,4]

35. A line passes through the points $(1,1)$ and $(6,4)$. Find an equation of the perpendicular line through $(5,-6)$.

$m = \frac{\Delta y}{\Delta x} = \frac{3}{5}$

$m_\perp = -\frac{5}{3}$

$y+6 = -\frac{5}{3}(x-5)$

OR

$y = -\frac{5}{3}x + \frac{7}{3}$

36. A line passes through the point $(-3,-2)$ and is parallel to the line described by $y = 8x - 7$. Find an equation of the line. Write your final answer in standard form.

$m = 8$

POINT $(-3,-2)$

$y+2 = 8(x+3)$

$y = 8x + 22$

$8x - y = -22$

37. A line passes through the points $(3,1)$ and $(3,0)$. Find equations of the lines parallel and perpendicular to the original line. Label which is which.

ORIGINAL LINE IS VERTICAL ... $x = 3$

PARALLEL ... $x = 7$

PERP ... $y = 0$

Objective: Find and apply the slope-intercept form of the equation of a line. [2,4]

38. Find the slope and y-intercept of the line described by $2x - 5y = 8$. Write your y-intercept as an ordered pair.

$2x - 8 = 5y$

$y = \frac{2}{5}x - \frac{8}{5}$

$m = \frac{2}{5}$

Y-INT $= (0, -\frac{8}{5})$

39. A line with slope $-3/7$ has y-intercept $(0,-4)$. Find an equation of the line. Write your final answer in standard form.

$y = -\frac{3}{7}x - 4$

$7y = -3x - 28$

$3x + 7y = -28$

40. A line is described by the equation $y = 3x-2$. Find the slope of the line, and determine two points on the line.

$m = 3$

TWO POINTS ... $(0,-2)$, $(1,1)$

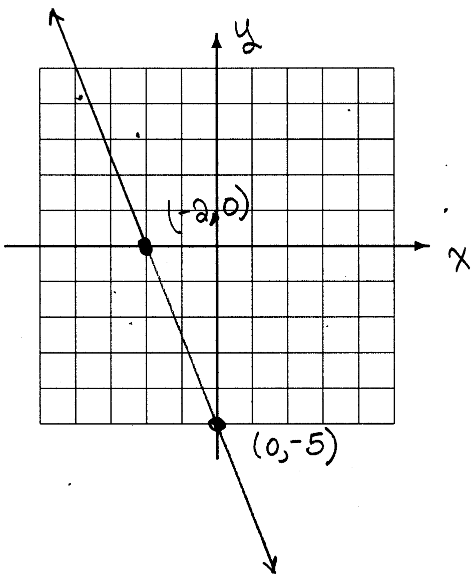
Objective: Graph a line using its slope and a point. [2,4]

41. A line is described by the equation $5x + 2y = -10$. Rewrite the equation is slope-intercept form. Then use the intercept and the slope to sketch the graph. Be sure to label your axes.

$2y = -5x - 10$

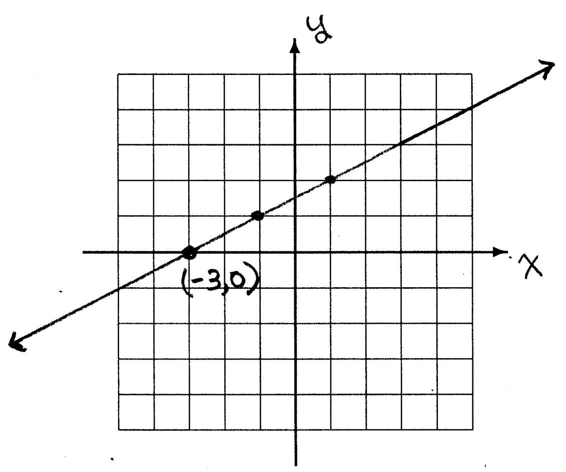
$y = -\frac{5}{2}x - 5$

$\frac{\Delta y}{\Delta x} = \frac{-5}{2} = \frac{\text{RISE}}{\text{RUN}}$



42. A line with slope $1/2$ passes through the point $(-3, 0)$. Sketch the graph of the line. Be sure to label your axes.

$\frac{\Delta y}{\Delta x} = \frac{1}{2} = \frac{\text{RISE}}{\text{RUN}}$



Objective: Apply lines and linear equations in real-world applications. [2,3,4]

43. The length of the humerus (the bone from the elbow to the shoulder) is a good indicator of height. A female with a humerus of length 26.1 cm is approximately 143.5 cm tall, while a female with a 20.4 cm humerus is about 127.6 cm tall. Assume that humerus length and height satisfy a linear equation. Determine that equation. Round all numbers to the nearest tenth.

$(26.1, 143.5)$

$(20.4, 127.6)$

$m = \frac{15.9}{5.7} \approx 2.8$

$y = 2.8x + b$

$127.6 = 2.8(20.4) + b$

$b \approx 70.5$ $\rightarrow$ $\boxed{y = 2.8x + 70.5}$

y = HEIGHT

x = HUMERUS LENGTH

44. A car currently worth \$24,575 depreciates at a constant rate of \$1752. Let v represent the value of the car in dollars, and let t represent time in years. Using the variables v and t, write an equation for the value of the car.

$V = -1752t + 24575$

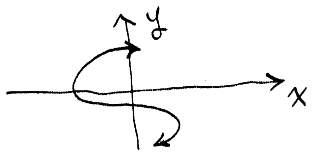
Objective: Determine whether a relation is a function. [10]

45. Carefully explain why this relation is not a function.

$$\{(1,2), (2,5), (3,8), (4,10), (-1,8), (3,9)\}$$

1ST COORD 3 HAS TWO DIFFERENT 2ND COORDS.

46. Sketch the graph of a relation that is not a function.



FAILS VERTICAL LINE TEST.

47. For any real number, x, let $f(x) = x^3 - x^2 + 1$. Does this define a function? How do you know?

Yes! A SINGLE X-INPUT GIVES A SINGLE $f(x)$ OUTPUT.

48. Does this table describe a function? How do you know?

x	-2	2	-5	8	7	13
y	1	1	1	1	1	1

Yes! No X-COORD IS REUSED.

Objective: Determine the domain and range of a function. [1]

49. What is the domain of the function $F(x) = x^2 + |x|$?

$(-\infty, \infty)$

ALL REAL #'S

50. What is the domain of the function $g(x) = \dfrac{x^2 + x - 6}{x^2 + 6x + 5}$?

$(x+1)(x+5) = 0$

$x = -1,\ x = -5$

ALL REAL #'S EXCEPT $x = -1$, $x = -5$

$(-\infty, -5) \cup (-5, -1) \cup (-1, \infty)$

51. What is the domain of the function $h(x) = \sqrt{2 - 4x}$?

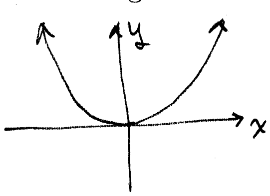
$2 - 4x \geq 0 \Rightarrow 2 \geq 4x$

$\frac{1}{2} \geq x$

ALL REAL X FOR WHICH $x \leq \frac{1}{2}$

$(-\infty, \frac{1}{2}]$

52. What is the range of the function $f(x) = x^2$?



$y \geq 0 \Rightarrow [0, \infty)$

Objective: Use function notation and evaluate functions. [*]

53. Let $f(x) = \sqrt[4]{x + 7}$. Evaluate $f(9)$. What about $f(-8)$?

$f(9) = \sqrt[4]{16} = \boxed{2}$

$f(-8) = \sqrt[4]{-1}$ NOT A REAL #

54. Let $g(y) = 2y^2 - 3y + 7$. Expand and simplify $g(t - 5)$.

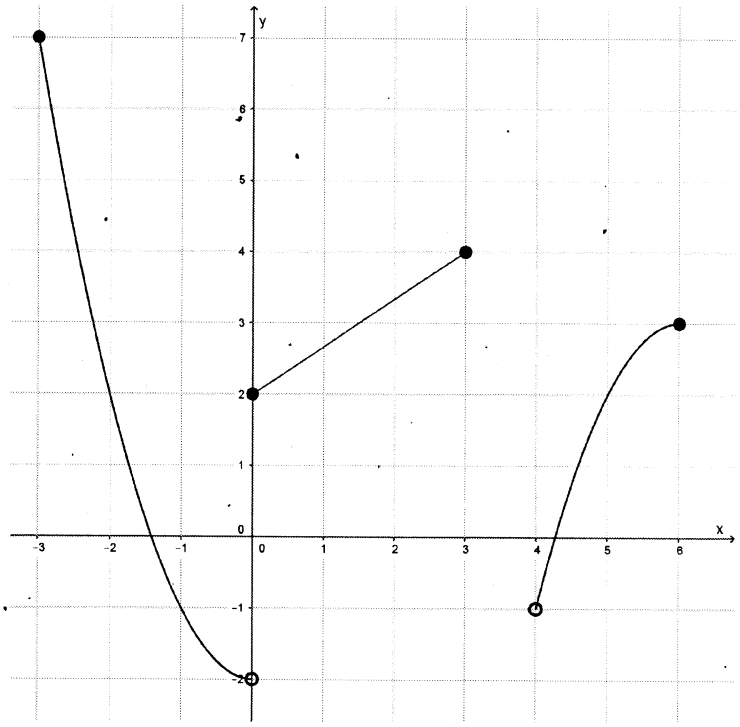
$g(t-5) = 2(t-5)^2 - 3(t-5) + 7$

$= 2t^2 - 20t + 50 - 3t + 15 + 7$

$= 2t^2 - 23t + 72$

Objective: Interpret graphs of functions. [1,10]

55. The graph of $y = h(x)$ is shown below. Use the graph to solve each part of this problem.



(a) Is this the graph of a function? How do you know?

YES, THE GRAPH PASSES THE VERTICAL LINE TEST.

(b) What is the domain of h?

$[-3,3] \cup (4,6]$

(c) What is the range of h?

$(-2,7]$

(d) Determine $h(-2)$.

$h(-2) \approx 2$

(e) Determine $h(3.5)$.

NOT DEFINED.

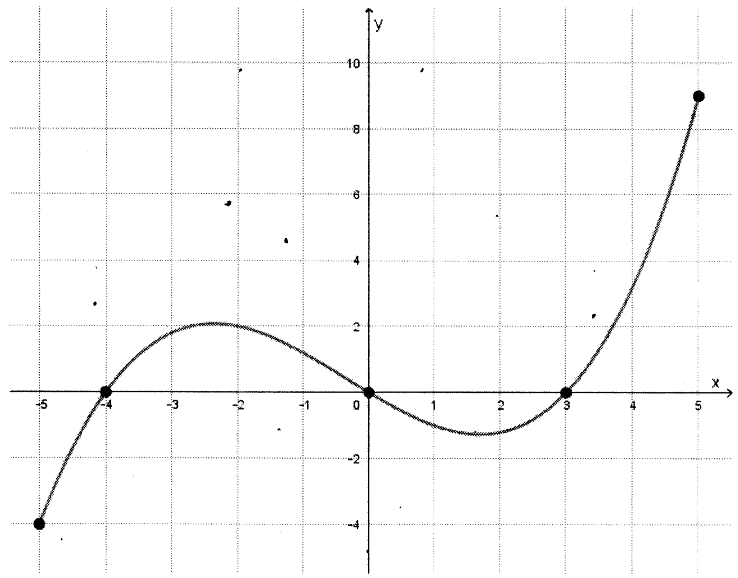
(f) Determine $h(0)$.

$h(0) = 2$

(g) Determine an x-value for which $h(x) = 3$. How many are there?

THERE ARE THREE. ONE IS $X = 6$.

56. The graph of $y = f(x)$ is shown below.



(a) What is the domain of f?

$[-5, 5]$

(b) What is the range of f?

$[-4, 9]$

(c) Determine intervals on which $f(x) < 0$.

$[-5, 4) \cup (0, 3)$

(d) Determine intervals on which $f(x) > 0$.

$(-4, 0) \cup (3, 5]$

Objective: Evaluate difference quotients. [5]

57. Evaluate the difference quotient $\frac{g(x+h)-g(x)}{h}$ for the function $g(x)=4x+1$. Completely expand and simplify your answer.

$$\frac{[4(x+h)+1]-[4x+1]}{h} = \frac{4x+4h+1-4x-1}{h} = \frac{4h}{h}$$

$$= \boxed{4}$$

58. Evaluate the difference quotient $\frac{f(x+h)-f(x)}{h}$ for the function $f(x)=x^2+x+2$. Completely expand and simplify your answer.

$$\frac{[(x+h)^2+(x+h)+2]-[x^2+x+2]}{h} = \frac{\cancel{x^2}+2xh+h^2+\cancel{x}+h+\cancel{2}-\cancel{x^2}-\cancel{x}-\cancel{2}}{h}$$

$$= \frac{2xh+h^2+h}{h} = \frac{\cancel{h}(2x+h+1)}{\cancel{h}} = \boxed{2x+h+1}$$

59. Evaluate the difference quotient $\frac{g(x+h)-g(x)}{h}$ for the function $g(x)=2x^2+6$. Completely expand and simplify your answer.

$$\frac{[2(x+h)^2+6]-[2x^2+6]}{h} = \frac{\cancel{2x^2}+4xh+2h^2+\cancel{6}-\cancel{2x^2}-\cancel{6}}{h}$$

$$= \frac{4xh+2h^2}{h} = \frac{\cancel{h}(4x+2h)}{\cancel{h}} = \boxed{4x+2h}$$