

**Math 129 - Review 4**  
May 9, 2021

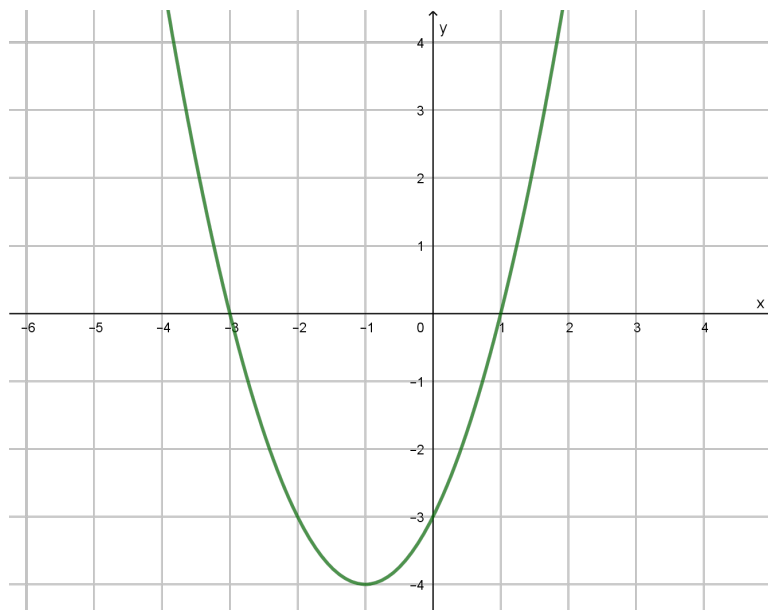
Name \_\_\_\_\_

These problems may help you review for the final exam. They are coded to match the course objectives from your syllabus. Unless otherwise indicated, you should simplify all answers by reducing fractions, simplifying radicals, and/or rationalizing denominators (as you've done on your ALEKS homework). Label your axes when graphing. **To adequately prepare for the comprehensive final exam, you should also study the earlier review packets, as well as old tests.**

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**Objective:** Find the vertex, intercepts, and symmetry axis of a parabola. [8]

1. The graph of the function  $f$  is the parabola shown below.



- (a) Is the leading coefficient of  $f(x)$  positive or negative? How do you know?
- (b) Determine the vertex of the parabola.
- (c) Determine the  $x$ -intercepts of the graph. Write them as ordered pairs.
- (d) Determine the  $y$ -intercept of the graph. Write it as an ordered pair.
- (e) Write an equation for the axis of symmetry of the parabola.

2. The graph of  $y = (x + 5)^2 - 8$  is a parabola. Where is its vertex? What is an equation of the symmetry axis?

3. A quadratic function has a leading coefficient of 3 and zeros  $x = 6$  and  $x = -2$ . Write its equation in standard form. What are the coordinates of the vertex of its graph?

4. The graph of  $f(x) = -2(x - 3)^2 + 4$  is a parabola.

(a) Does the parabola open up or down? How do you know?

(b) Determine the vertex of the parabola.

(c) Write an equation for the symmetry axis.

(d) Determine the  $x$ -intercepts of the parabola.

**Objective:** Write a quadratic function in vertex form. [6,8]

5. Write the quadratic function  $f(x) = x^2 + 6x + 11$  in vertex form.

6. Write the quadratic function  $g(x) = x^2 - 4x - 2$  in vertex form.

7. Write the quadratic function  $h(x) = 2x^2 + 16x - 24$  in vertex form.

**Objective:** Solve application problems involving quadratic functions and parabolas. [8]

8. A ball is thrown upward with a velocity of 48 feet per second from the top of a 144-foot building. Its height, in feet, at time  $t$  (in seconds) is given by  $h(t) = -16t^2 + 48t + 144$ . What is the maximum height of the ball? How long until the ball hits the ground?

9. The total cost of manufacturing a set of golf clubs is given by

$$C(x) = 800 - 10x + 0.20x^2,$$

where  $x$  is the number of sets of golf clubs produced. How many sets of golf clubs should be manufactured to incur minimum cost and what is that minimum cost?

10. Cindy wants to construct two side-by-side dog-training pens of the same size and sharing one common side. She has 400 ft of fencing material to use. What values of  $x$  and  $y$  maximize the combined areas of the pens?

**Objective:** Find the zeros of a polynomial and determine their multiplicities. [12]

11. Find the zeros of  $f(x) = x^2 + 3x + 2$ .

12. Find the zeros of  $g(x) = 4(x - 1)^3(x + 5)(x + 8)^2$  and state their multiplicities.

13. Let  $f(x) = x^7(x - 1)^5(x + 2)$ . Find the zeros of  $f$  and state their multiplicities.

14. Let  $p(x) = x^5 - 4x^4 + 4x^3$ . Find the zeros of  $p$  and state their multiplicities.

**Objective:** Carry out polynomial long division and synthetic division. [12]

15. Use long division to divide:  $\frac{3x^2 + 3x - 14}{x - 2}$

16. Use long division to divide:  $(6x^3 - 5x^2 - 3) \div (3x + 2)$

17. Use long division to divide:  $(8x^3 - 6x^2 - 11x + 13) \div (2x^2 - x)$

18. Use long division to divide:  $(-11 + 12x^2 + 5x + 9x^3) \div (-3x^2 - 2x + 2)$

19. Use synthetic division to divide:  $\frac{3x^2 + 3x - 14}{x - 2}$

20. Use synthetic division to divide:  $(3x^4 + 7x^3 - 5x^2 + x - 6) \div (x + 3)$

21. Use synthetic division to divide:  $\frac{x^3 - 21x + 20}{x + 5}$

**Objective:** Apply the remainder and factor theorems. [12]

22. Use synthetic division and the remainder theorem to evaluate  $P(-2)$  if  $P(x) = -2x^4 - 2x^3 + x + 3$ .

23. Use synthetic division and the remainder theorem to evaluate  $f(2)$  if  $f(x) = 2x^2 + 3x + 1$ .

24. Let  $f(x) = x^3 - 2x^2 - 4x + 8$ . Evaluate  $f(2)$ . Based on the value of  $f(2)$ , determine whether  $x - 2$  is a factor of  $f$ .

25. Let  $f(x) = 4x^3 - 3x^2 - 2x + 6$ . Evaluate  $f(-1)$ . Based on the value of  $f(-1)$ , determine whether  $x + 1$  is a factor of  $f$ .

26. The only zeros of a polynomial are  $x = 0$ ,  $x = 5$  and  $x = -8$ . Determine the factors of the polynomial.



**Objective:** Determine the end behavior of a polynomial function. [12,13]

27. Describe the end behavior of the graph of  $f(x) = -4x^8 - 19x^5 + 52x^2 - 17x + 100$ .

28. Describe the end behavior of the graph of  $f(x) = x^3 - 3x^2 - 9x - 17$ .

29. Describe the end behavior of the graph of  $f(x) = -3x^2(x + 1)(x^2 + 1)$ .

**Objective:** Use multiplicities of zeros and end behavior to graph a polynomial function.  
[12,13]

30. Consider the polynomial  $f(x) = -x(x - 2)^3(2x + 1)^2$ .

(a) Determine the degree of  $f$  and the leading coefficient.

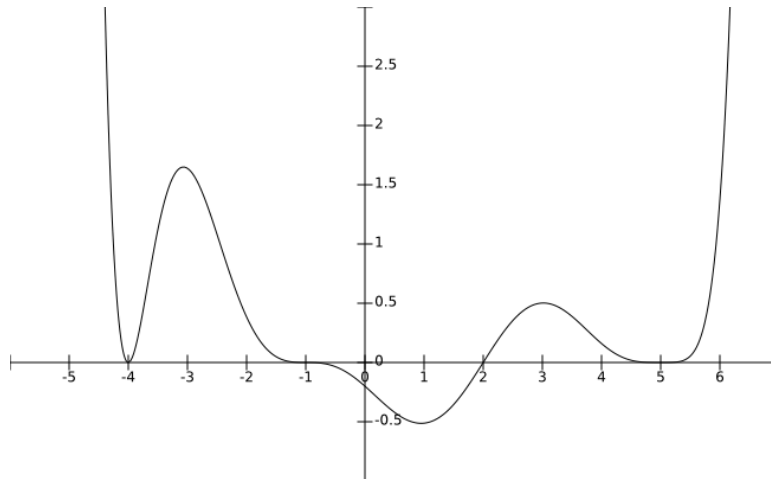
(b) State the zeros of  $f$  and their corresponding multiplicities.

(c) Describe the end behavior of the graph of  $f$ .

(d) Determine the  $y$ -intercept.

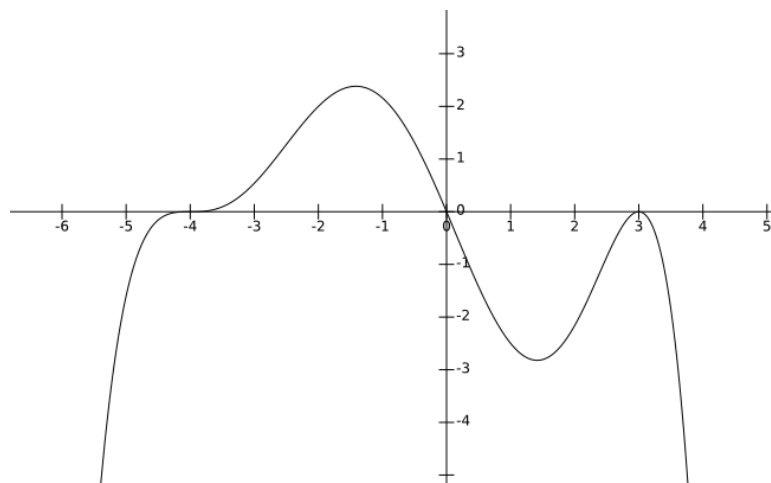
(e) Roughly sketch the graph of  $f$ . Be sure that your graph correctly illustrates the  $y$ -intercept, the end behavior, and the behavior near the  $x$ -intercepts.

31. The graph of a polynomial is shown below.



- (a) Is the degree even or odd?
- (b) Is the leading coefficient positive or negative?
- (c) Which zeros have multiplicity one?
- (d) Which zeros have even multiplicity?
- (e) Which zeros have odd multiplicity greater than 1?

32. Give the factored form of a polynomial whose graph has the same general shape of the one given below.



**Objective:** Determine the domain of a rational function. [1,13]

33. Determine the domain of the function  $f(x) = \frac{1}{x^2 + 5x - 6}$ .

34. Determine the domain of the function  $f(x) = \frac{x + 4}{2x^2 + 11x + 12}$ .

**Objective:** Determine the vertical, horizontal, and/or slant asymptotes of the graph of a rational function. [13]

35. Let  $f(x) = \frac{x^2 + 1}{(x - 3)(x + 2)}$ . Determine the vertical asymptotes of the graph of  $f$ .

36. Let  $g(x) = \frac{2x - 2}{(x - 1)(x - 5)}$ . Determine the vertical asymptotes of the graph of  $g$ .

37. Let  $f(x) = \frac{x^3 + 3x^2 + 1}{x^2 - x}$ . Determine the slant asymptote and the vertical asymptotes of the graph of  $f$ .

38. Determine the horizontal asymptote of the graph of  $R(x) = \frac{3x^2 + 2x - 9}{2x^2 - x - 1}$ .
39. Determine the horizontal asymptote of the graph of  $H(x) = \frac{3x}{6x^2 + 3x + 112}$ .
40. Explain how you know that the graph of  $f(x) = \frac{x^3 - 8}{x + 7}$  has no horizontal asymptote.
41. Give an example of a rational function whose graph has the horizontal asymptote  $y = 3$  and vertical asymptotes  $x = 5$  and  $x = -9$ .

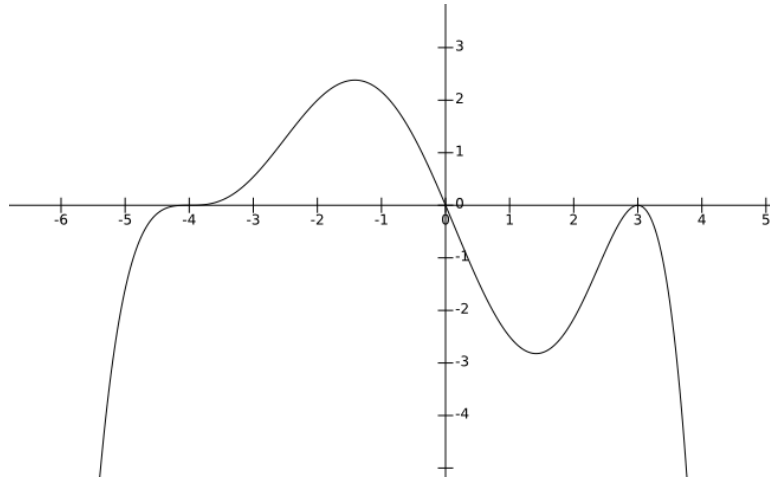
**Objective:** Sketch the graph of a rational function. [13]

42. Sketch the graph of  $y = \frac{2x - 4}{x - 5}$ .

43. Sketch the graph of  $y = \frac{x^2 - 2x - 3}{2x + 6}$ .

**Objective:** Solve polynomial inequalities. [11,12]

44. Let  $p(x)$  be the polynomial whose graph is shown below. Use the graph to solve the inequality  $p(x) \geq 0$ .



45. Solve and graph the solution set on a number line:  $(x + 4)(x + 9)(x - 5) \geq 0$

46. Solve and graph the solution set on a number line:  $x^3 + 5x^2 \leq 4x + 20$

**Objective:** Solve rational inequalities. [11,12,13]

47. Solve and graph the solution set on a number line:  $f(x) = \frac{-x^2 - 9x}{x^2 + 9x + 18} \geq 0$

48. Solve and graph the solution set on a number line:  $f(x) = \frac{x - 1}{x + 5} < 0$

49. Solve and graph the solution set on a number line:  $\frac{-1}{x - 6} < \frac{2}{9 - x}$