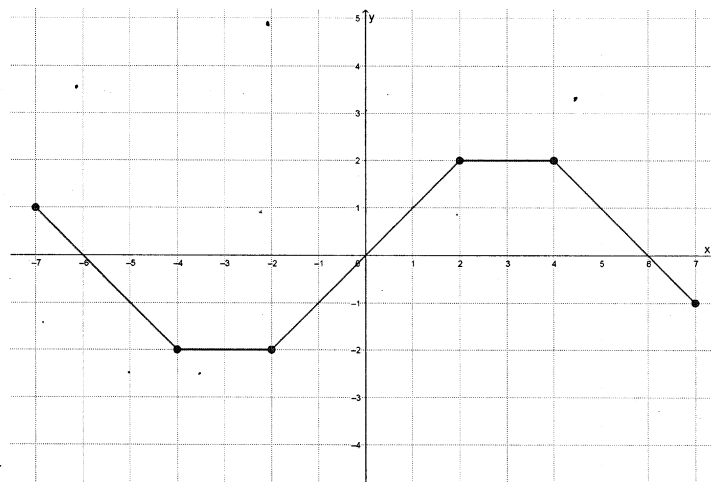# Math 129 - Test 3
April 22, 2021

Name __key__

Score _____

Show all work to receive full credit. Supply explanations where necessary. Label your axes when graphing. You may get partial credit on multiple choice problems if you show correct work or explanations.

1. (10 points [8,10])   The graph of the function f is shown below.

(a) Find the x-intercepts.

$$(-6,0), \ (0,0), \ (6,0)$$

(b) Determine the interval(s) on which f is constant.

$$(-4,-2) \cup (2,4)$$

(c) Determine the interval(s) on which f is increasing.

$$(-2,2)$$

(d) Is f an even function, an odd function, or neither? Explain how you know.

f IS ODD -- ITS GRAPH IS SYMMETRIC ABOUT THE ORIGIN.

(e) Is f continuous on its domain? Explain how you know.

YES, THE ENTIRE GRAPH IS CONSTRUCTED WITH NO JUMPS OR BREAKS. GRAPH CAN BE DRAWN WITHOUT PICKING UP THE PEN.

1

2. (8 points [1,5]) Consider the following function.

$$f(x) = \begin{cases} \sqrt[3]{x}, & x \le 8 \\ x^2 + 1, & x > 9 \end{cases}$$

(a) Evaluate $f(8)$.

$$f(8) = \sqrt[3]{8} = \boxed{2}$$

(b) Evaluate $f(0)$.

$$f(0) = \sqrt[3]{0} = \boxed{0}$$

(c) Evaluate $f(9)$.

$$f(9) \text{ IS NOT DEFINED.}$$

(d) What is the domain of f?

$$(-\infty, 8] \cup (9, \infty)$$

3. (4 points [5]) The temperature T in degrees Fahrenheit in terms of the Celsius temperature d is given by $T = \frac{9}{5}d + 32$.

The temperature C in degrees Celsius in terms of the Kelvin temperature v is given by $C = v - 273$.

Write a formula for the temperature F in degrees Fahrenheit in terms of the Kelvin temperature v.

$$T = \frac{9}{5}d + 32$$

$$C = \boxed{v - 273}$$

$$F = \frac{9}{5}\left(v - 273\right) + 32$$

OR

$$F = \frac{9}{5}v - \frac{2297}{5}$$

OR

$$F = 1.8v - 459.4$$

2

4. (8 points [5]) The functions f and g are defined in the table shown below. Use the data from the table to evaluate each of the following.

x	1	2	3	4	5
$f(x)$	0	2	-5	6	2
$g(x)$	1	1	4	3	8

(a) $(f - g)(4)$

$$f(4) - g(4) = 6 - 3 = \boxed{3}$$

(b) $(fg)(3)$

$$f(3) \times g(3) = -5 \times 4 = \boxed{-20}$$

(c) $\left(\dfrac{g}{f}\right)(2)$

$$\frac{g(2)}{f(2)} = \boxed{\frac{1}{2}}$$

(d) $(g \circ f)(5)$

$$g(f(5)) = g(2) = \boxed{1}$$

5. (8 points [5]) Refer to the functions f and g defined in the problem above.

(a) Complete the table for the function $f \circ g$. Use DNE if the value does not exist (or is not defined).

$f(g(x))$

x	1	2	3	4	5
$(f \circ g)(x)$	0	0	6	-5	DNE

(b) What is the domain of $(f \circ g)$?

$$\{1, 2, 3, 4\}$$

(c) What is the range of $(f \circ g)$?

$$\{0, 6, -5\}$$

6. (10 points [9]) Think about the function $f(x) = x^2$, which is one of our basic toolbox functions.

(a) List any three important properties of the function.

① DOMAIN = $(-\infty, \infty)$

② RANGE = $[0, \infty)$

③ EVEN FUNCTION / GRAPH SYMMETRIC ABOUT y-AXIS

④ U-SHAPE GRAPH. VERTEX AT $(0, 0)$

⑤ MIN VALUE IS ZERO AT $X = 0$

⑥ DECREASING ON $(-\infty, 0)$, INCREASING ON $(0, \infty)$.

(b) Carefully explain how the graph of $g(x) = (x+3)^2 - 5$ can be obtained from the graph of f.
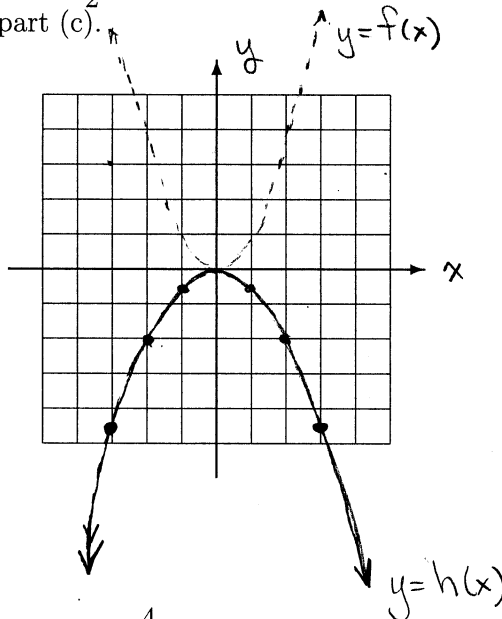
START WITH THE GRAPH OF f, THEN

① SHIFT 3 UNITS LEFT

② SHIFT 5 UNITS DOWN

(c) Carefully explain how the graph of $h(x) = -\dfrac{1}{2}x^2$ can be obtained from the graph of f.

START WITH THE GRAPH OF f, THEN

① VERTICALLY COMPRESS IT SO IT IS $\frac{1}{2}$ AS FAR FROM X-AXIS

② FLIP (OR REFLECT) ABOUT THE X-AXIS

(d) Sketch the graph of $h(x) = -\dfrac{1}{2}x^2$. Label your axes, and be sure your graph does what you described in part (c).
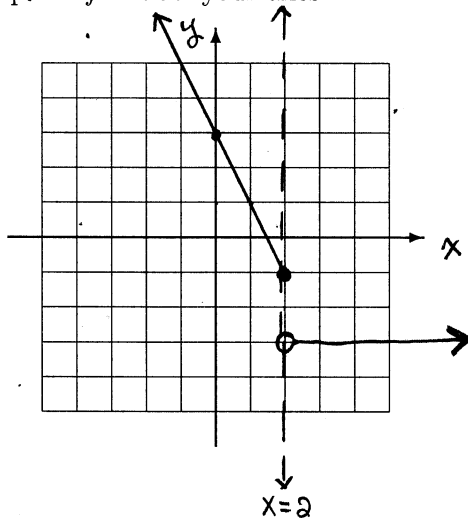

4

7. (8 points [1,5,9]) Consider the following function.

$$f(x) = \begin{cases} -2x + 3, & x \leq 2 \\ -3, & x > 2 \end{cases}$$

(a) What is the domain of f?

$$(-\infty, 2] \cup (2, \infty) = \boxed{(-\infty, \infty)}$$

(b) Carefully sketch the graph of f. Label your axes.

$x = 2$

(c) Is f a continuous function? If not, where is it discontinuous?
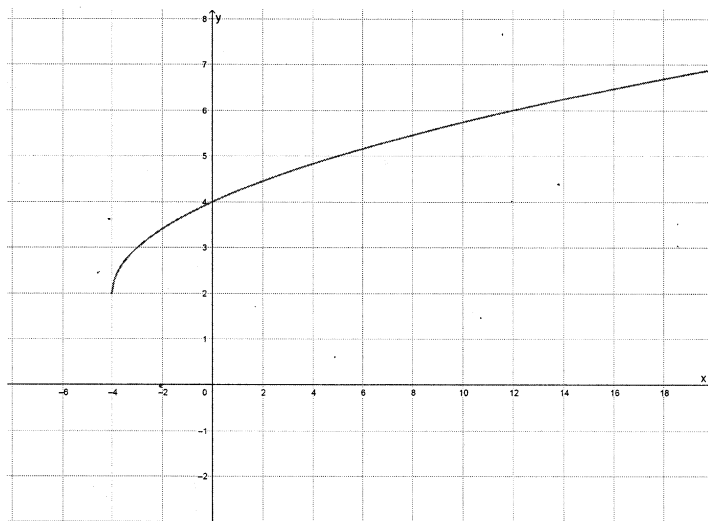
No, IT IS DISCONTINUOUS AT $X = 2$.

8. (4 points [9]) The graph of $y = \sqrt{x}$ is shifted to obtain the new graph shown below. What is an equation for the new graph?

LEFT 4

Up 2

$\downarrow$

$$\boxed{y = \sqrt{x+4} + 2}$$

5

9. (3 points [5]) Find two functions f and g so that $(f \circ g)(x) = (3x^2 + 5)^7$.

$$f(x) = (x)^7 \qquad g(x) = 3x^2 + 5$$

10. (7 points [5]) Let $f(x) = \dfrac{5}{x+3}$ and $g(x) = \dfrac{6}{x-7}$.

(a) Compute $(f + \cdot g)(2)$. Write your answer as a fraction in lowest terms.

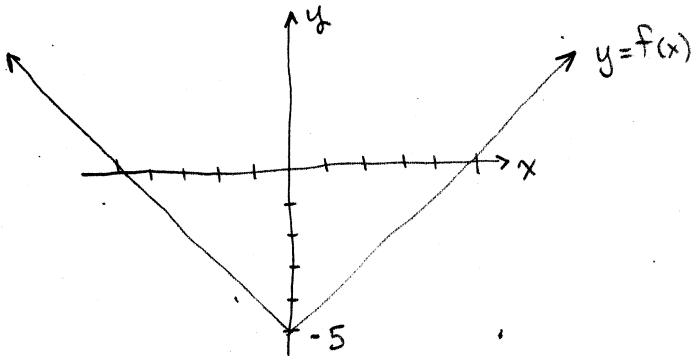$$f(2) + g(2) = \frac{5}{5} + \frac{6}{-5} = \boxed{-\frac{1}{5}}$$

(b) Compute $\left(\dfrac{f}{g}\right)(0)$. Write your answer as a fraction in lowest terms.

$$\frac{f(0)}{g(0)} = \frac{5/3}{-6/7} = \boxed{-\frac{35}{18}}$$

(c) Find and simplify (as much as possible) the formula for $(f \circ g)(x)$.

$$f(g(x)) = \frac{5}{\frac{6}{x-7} + 3} \cdot \frac{x-7}{x-7} = \frac{5(x-7)}{6 + 3(x-7)}$$

$$= \boxed{\frac{5x - 35}{3x - 15}}$$

11. (2 points [9]) What is the range of the function $f(x) = |x| - 5$? Write your answer in interval notation.

$y = f(x)$

RANGE IS

$$[-5, \infty)$$

6

12. (18 points [8,10]) Consider the following types of functions:

A – Constant functions, $f(x) = c$

B – Linear functions (Not constant), $f(x) = mx + b$, $m \neq 0$

C – Absolute value function, $f(x) = |x|$

D – Squaring function, $f(x) = x^2$

E – Square root function, $f(x) = \sqrt{x}$

F – Cubing function, $f(x) = x^3$

G – Cube root function, $f(x) = \sqrt[3]{x}$

For each part below, indicate (by letter) which functions have the given property. List all that apply, or say "None" if none apply.

(a) The graph is a line.

$$A, B$$

(b) The graph is symmetric about the x-axis.

$$\text{None}$$

(c) The domain is the set of all real numbers.

$$A, B, C, D, F, G$$

(d) The function has a minimum value.

$$A, C, D, E$$

(e) The graph is symmetric about the origin.

$$F, G$$

(f) The graph has V shape.

$$C$$

(g) The function is even.

$$A, C, D$$

(h) The range is the set of all real numbers.

$$B, F, G$$

(i) The graph has a U shape.

$$D$$

13. (2 points [8,10]) Which one of the following is true about the graph of an even function?

   (a) The graph is symmetric about the origin.
   (b) The graph is symmetric about the x-axis.
   (c) The graph is symmetric about the y-axis.
   (d) The graph is symmetric about its oblique asymptote.

14. (2 points [5]) Find two functions f and g so that $(f \circ g) = \sqrt{5x + 3}$.

   (a) $f(x) = \sqrt{x}$, $g(x) = 5x + 3$
   (b) $f(x) = 5x + 3$, $g(x) = \sqrt{x}$
   (c) $f(x) = \sqrt{x + 3}$, $g(x) = 5x$
   (d) Both (a) and (c).

15. (2 points [8,10]) Which one of the following is true about the graph of an odd function?

   (a) The graph is symmetric about the origin.
   (b) The graph is symmetric about the x-axis.
   (c) The graph is symmetric about the y-axis.
   (d) The graph has an odd number of pieces.

16. (2 points [8,10]) Which function has a graph that is symmetric about the y-axis?

   (a) $g(x) = 0.0000001x^3$
   (b) $f(x) = 8723 - 5135x^2$
   (c) $h(x) = \sqrt[3]{x}$
   (d) Both (a) and (c).

17. (2 points [5]) For functions f and g, what does the expression $(f+g)(x) = f(x)+g(x)$ mean?

   (a) It describes the distributive property.
   (b) It describes the composition of functions.
   (c) It defines function addition.
   (d) Both (a) and (b).