Math 129 - Final Exam

May 13, 2021

Name key

Score

Show all work to receive full credit. Supply explanations where necessary. Label your axes when graphing.

1. (3 points [3]) Solve for y: $-2(y+7)+28 > 2(6-y)$

$$-2y - 14 + 28 > 12 - 2y$$

$$-2y + 14 > 12 - 2y$$

$$14 > 12$$

ALWAYS TRUE.

ALL #'S ARE SOLUTIONS.

2. (7 points [11]) Solve for w. Write your solution set in interval notation, and graph it on a number line.

$$|10-4w| \leq 6$$

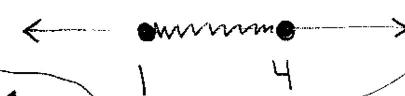
$$-6 \leq 10 - 4w \leq 6$$

$$-16 \leq -4w \leq -4$$

$$\underbrace{4 \geq w \geq 1}$$

$$1 \leq w \leq 4$$

[1, 4]



3. (6 points [7]) Solve for x. After finding the exact solutions, write them in decimal form, rounded to the nearest hundredth.

$$3x^2 + x - 1 = 0$$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(3)(-1)}}{2(3)}$$

$$= \frac{-1 \pm \sqrt{13}}{6}$$

$$\frac{-1+\sqrt{13}}{6} \approx 0.43$$

$$\frac{-1-\sqrt{13}}{6} \approx -0.77$$

4. (6 points [3,11]) Solve for x: $\frac{x}{2} = \frac{2}{x+3}$

$$x(x+3) = 2 \cdot 2$$
$$x^2 + 3x = 4$$
$$x^2 + 3x - 4 = 0$$
$$(x-1)(x+4) = 0$$

$$\boxed{x = 1, \quad x = -4}$$

5. (4 points [11]) Solve for x: $\sqrt[3]{2x-5} + 4 = 2$

$$\sqrt[3]{2x-5} = -2$$
$$2x - 5 = -8$$
$$2x = -3$$
$$\boxed{x = -\frac{3}{2}}$$

6. (5 points [3]) Find the x- and y-intercepts of the line described by $7y = 3x - 12$.

X-INT:

$y = 0 \Rightarrow \quad 0 = 3x - 12$

$x = 4$

$\boxed{(4, 0)}$

Y-INT:

$x = 0 \Rightarrow \quad 7y = -12$

$y = \frac{-12}{7}$

$\boxed{\left(0, -\frac{12}{7}\right)}$

7. (5 points [2,4]) Find an equation of the line that passes through the points $(3, 1)$ and $(-5, 7)$. Write your final answer in slope-intercept form.

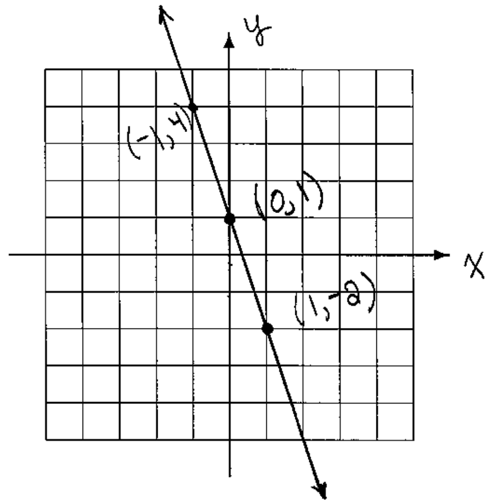
$$m = \frac{1-7}{3+5} = \frac{-6}{8} = -\frac{3}{4}$$

$$y - 1 = -\frac{3}{4}(x-3)$$

$$y = -\frac{3}{4}x + \frac{9}{4} + 1$$

$$\boxed{y = -\frac{3}{4}x + \frac{13}{4}}$$

8. (5 points [2,4]) A line with slope -3 passes through the point $(1,-2)$. Graph the line and label the coordinates of two points on the line. (Also label your axes.)



9. (6 points [2,4]) The line W passes through the point $(4,7)$ and is perpendicular to the line given by $y-1=\frac{5}{2}(x+1)$. Find an equation for the line W. Write your final answer in standard form ($Ax+By=C$).

$m = \frac{5}{2} \Rightarrow m_\perp = -\frac{2}{5}$

$y-7 = -\frac{2}{5}(x-4)$

$y-7 = -\frac{2}{5}x + \frac{8}{5}$

$\frac{2}{5}x + y = \frac{8}{5} + 7$

$\boxed{\frac{2}{5}x + y = \frac{43}{5}}$

10. (4 points [1]) Determine the domain and range of $f(x)=\sqrt{x-1}+3$.

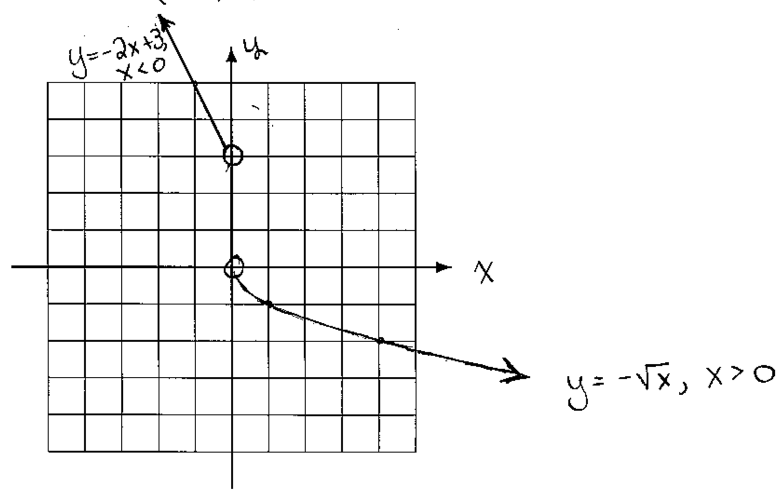
Domain: $x-1 \geq 0 \Rightarrow x \geq 1$

$\boxed{[1,\infty)}$

Range: $f(x) \geq 3 \Rightarrow \boxed{[3,\infty)}$

11. (6 points [1,5,9]) Carefully sketch the graph of f. Label your axes.

$$f(x) = \begin{cases} -2x+3, & x<0 \\ -\sqrt{x}, & x>0 \end{cases}$$



12. (4 points [13]) Determine the vertical asymptote(s) of the graph of $R(x) = \dfrac{2x-2}{(x+3)(x-1)}$.

$$\frac{2\cancel{(x-1)}}{(x+3)\cancel{(x-1)}} = \frac{2}{x+3}, \quad x \neq 1$$

ZERO DENOM / NONZERO NUMER

WHEN $\boxed{x = -3}$

13. (5 points [5]) Let $f(x) = 2x^2 + 3x$ and $g(x) = \sqrt{4x}$. Compute $(g \circ f)(-3)$.

$f(-3) = 2(9) - 9$
$= 9$

$$g(f(-3)) = g(9) = \sqrt{36} = \boxed{6}$$

14. (12 points [8,9]) The graph of $f(x) = (x - 120)^2 + 100$ is a parabola.

(a) Explain how the graph of f can be obtained from the graph of $y = x^2$.

① SHIFT RIGHT 120 UNITS

② SHIFT UP 100 UNITS

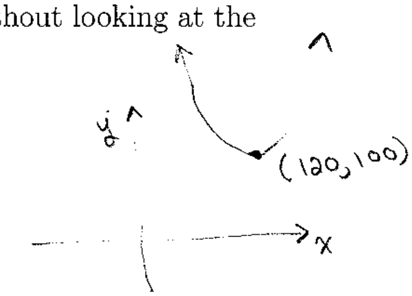
(b) Determine the vertex and an equation for the axis of symmetry of the graph of f.

VERTEX (120, 100)

AXIS OF SYMMETRY : X = 120

(c) Explain how you know the graph has no x-intercepts even without looking at the graph.

THE PARABOLA OPENS UPWARD,
AND THE VERTEX, (120,100),
LIES ABOVE THE X-AXIS,
THE GRAPH CANNOT CROSS THE X-AXIS.



15. (6 points [12]) Use synthetic division and the remainder theorem to evaluate $p(4)$ when $p(x) = 2x^2 - 4x + 7$.

4	2	-4	7
		8	16
	2	4	23

$\boxed{p(4) = 23}$

16. (16 points [11,12,13]) Consider the polynomial $f(x) = 5(x-7)(x-1)^2(x+4)^3$.

(a) Determine the leading term and degree of f.

Leading term $= 5x\,x^2\,x^3 = \boxed{5x^6}$

Degree is $\boxed{6}$

(b) State the zeros of f and their corresponding multiplicities.

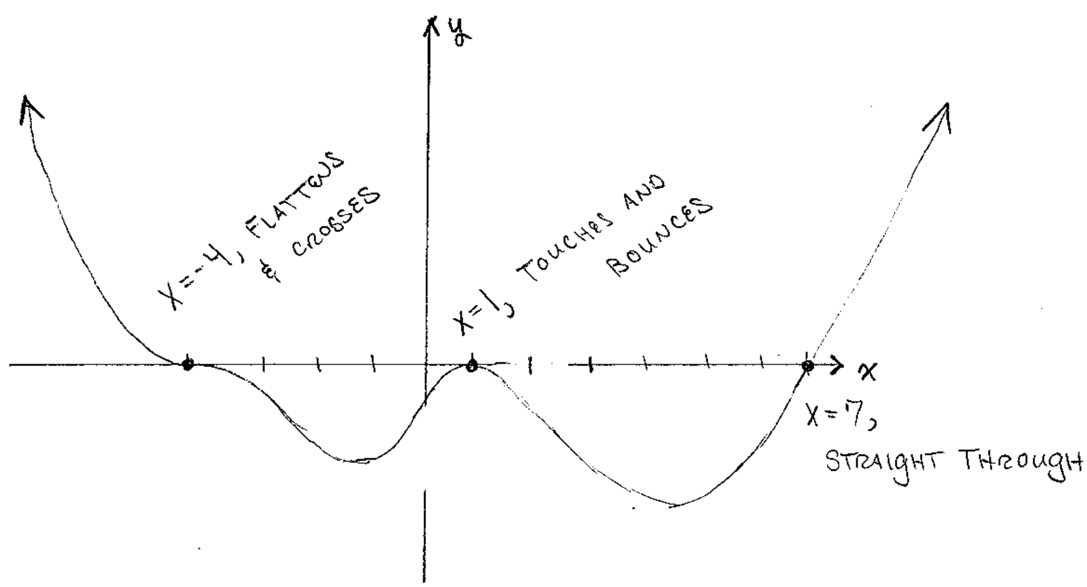
$x=7$ mult 1, $x=1$ mult 2, $x=-4$ mult 3

(c) Describe the end behavior of the graph of f.

Even degree

& positive leading coeff $\Rightarrow$ Up on both ends

(d) Roughly sketch the graph of f. Be sure that your graph correctly illustrates the end behavior and the behavior at the x-intercepts.



(e) Use your graph to solve $f(x) \le 0$. Write your answer in interval notation.

On or below x-axis on $\boxed{[-4, 7]}$.