Math 130 - Quiz 12

December 4, 2019

Name key

Score

Show all work to receive full credit. Supply explanations when necessary.

1. (8 points) Solve each equation.

   (a) $2^{3x-7} = 32 = 2^5$

   $3x - 7 = 5$

   $3x = 12 \Rightarrow \boxed{x = 4}$

   (b) $3^x = 2^{x-1}$

   $\ln 3^x = \ln 2^{x-1}$

   $\underbrace{x \ln 3 = (x-1)\ln 2 = x \ln 2 - \ln 2}$

   $x \ln 3 - x \ln 2 = -\ln 2 \Rightarrow x(\ln 3 - \ln 2) = -\ln 2$

   $x = \dfrac{-\ln 2}{\ln 3 - \ln 2} \approx 1.7095$

   (c) $\log x + \log(x+3) = 1$

   $\log x(x+3) = 1$

   $x(x+3) = 10$

   $x^2 + 3x - 10 = 0$

   $(x+5)(x-2) = 0$

   $\cancel{x = -5}$, $\boxed{x = 2}$

   (d) $e^{2x} - 9e^x + 14 = 0$

   $u = e^x$

   $u^2 - 9u + 14 = 0$

   $(u-7)(u-2) = 0$

   $u = 7$ or $u = 2$

   $e^x = 7 \Rightarrow x = \ln 7$

   $e^x = 2 \Rightarrow x = \ln 2$

Turn over.

2. (4 points) In 1997, the population of the United States was about 266 million, and it was growing exponentially at 0.9% per year. Find a model of the form $P(t) = P_0 e^{kt}$ that describes the population at time t. What does your model predict the U.S. population will be in 2020?

$t=0$ REPRESENTS 1997

$P(0) = P_0 = 266$ (IN MILLIONS)

$P(1) = 266 + 0.009(266) = 268.394$ (IN MILLIONS)

$= 266e^k$

$1.009 = e^k$

$k = \ln 1.009$

$$\boxed{P(t) = 266e^{t \ln 1.009}}$$

IN 2020...

$P(23) = 266e^{23 \ln 1.009}$

$\approx$ 326.9 MILLION

3. (3 points) One-hundred animals were released into a preserve where their population grows according to the model $P(t) = \dfrac{1000}{1+9e^{-0.1656t}}$, where t is measured in months. After how long will the population reach 999 animals? Will the population ever reach 1000?

$$\frac{1000}{1+9e^{-0.1656t}} = 999 \Rightarrow 1+9e^{-0.1656t} = \frac{1000}{999}$$

$$e^{-0.1656t} = \frac{1}{9}\left(\frac{1000}{999} - 1\right)$$

$$\approx 1.112223 \times 10^{-4}$$

$$-0.1656t = \ln\left[\frac{1}{9}\left(\frac{1000}{999} - 1\right)\right]$$

$t \approx$ 55 MONTHS

FOR THE POPULTION TO REACH 1000,

WE MUST HAVE

$$1+9e^{-0.1656t} = 1$$

OR

$$9e^{-0.1656t} = 0$$

NOT POSSIBLE. $P(t) < 1000$ ALWAYS!