

# Math 130 - Quiz 8 TH

October 23, 2019

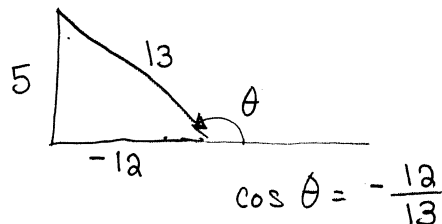
Name key

Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations when necessary. You must work individually on this quiz. This quiz is due October 28.

IF  $\theta$  IS  
IN 2<sup>ND</sup>,  
 $\theta/2$  IS IN  
1<sup>ST</sup>

1. (3 points) Given that  $\theta$  is a 2nd quadrant angle with  $\sin \theta = 5/13$ , determine the exact values of  $\sin(\theta/2)$  and  $\cos(\theta/2)$ .



$$\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}} = \sqrt{\frac{1 + 12/13}{2}} = \sqrt{\frac{25/13}{2}} = \frac{5}{\sqrt{26}}$$

$$\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}} = \sqrt{\frac{1 - 12/13}{2}} = \sqrt{\frac{1/13}{2}} = \frac{1}{\sqrt{26}}$$

2. (3 points) Find the solutions in the interval  $[0, 2\pi)$ :  $\cos 2x - \cos 6x = 0$

$$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$-2 \sin(4x) \sin(-2x) = 0$$

$$2 \sin 4x \sin 2x = 0$$

$$\sin 4x = 0 \text{ or } \sin 2x = 0$$

$$4x = 0 + 2k\pi,$$

$$2x = 0 + 2k\pi,$$

$$\pi + 2k\pi$$

$$\pi + 2k\pi$$

$$x = \frac{2k\pi}{4}, \frac{\pi + 2k\pi}{4}$$

$$x = \frac{2k\pi}{2}, \frac{\pi + 2k\pi}{2}$$

3. (3 points) Verify the trigonometric identity:  $(\sin x + \cos x)^2 = 1 + \sin 2x$

$$(\sin x + \cos x)^2 = \sin^2 x + 2 \sin x \cos x + \cos^2 x$$

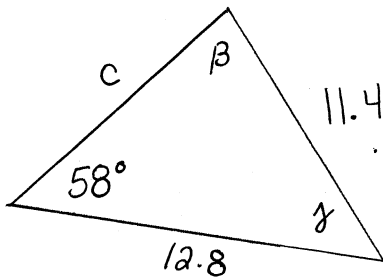
$$= \sin^2 x + \cos^2 x + 2 \sin x \cos x$$

$$= 1 + \sin 2x \quad \checkmark$$

Turn over.

4. (6 points) Solve the triangle. If two solutions exist, find both.

$$\alpha = 58^\circ, \quad a = 11.4, \quad b = 12.8$$



$$\frac{\sin 58^\circ}{11.4} = \frac{\sin \beta}{12.8}$$

$$\sin \beta = \frac{12.8 \sin 58^\circ}{11.4} \approx 0.9522$$

$$\beta \approx 72.21^\circ \text{ or } 107.79^\circ$$

$$\beta \approx 72.21^\circ$$

$$\begin{aligned} \gamma &\approx 180^\circ - (72.21^\circ + 58^\circ) \\ &= 49.79^\circ \end{aligned}$$

$$\beta \approx 107.79^\circ$$

$$\begin{aligned} \gamma &\approx 180^\circ - (107.79^\circ + 58^\circ) \\ &= 14.21^\circ \end{aligned}$$

$$\frac{\sin 58^\circ}{11.4} = \frac{\sin 49.79^\circ}{c}$$

$$c \approx \frac{11.4 \sin 49.79^\circ}{\sin 58^\circ} \approx 10.27$$

$$\frac{\sin 58^\circ}{11.4} = \frac{\sin 14.21^\circ}{c}$$

$$\begin{aligned} c &= \frac{11.4 \sin 14.21^\circ}{\sin 58^\circ} \\ &\approx 3.30 \end{aligned}$$

$$\beta \approx 72.21^\circ$$

$$\gamma \approx 49.79^\circ$$

$$c \approx 10.27$$

$$\beta \approx 107.79^\circ$$

$$\gamma \approx 14.21^\circ$$

$$c = 3.30$$