Math 130 - Review 2
October 7, 2019

Name key

Score

These problems may help you review for Test 2. Your actual test will not be as long as this review packet. When providing exact answers, simplify as much as possible.

1. At what x-values (give all of them) does the graph of $y = \sec x$ have vertical asymptotes?

THE GRAPH OF $y = \sec x$ HAS VERTICAL ASYMPTOTES AT THE ZEROS OF $y = \cos x$, THAT IS AT ALL ODD MULTIPLES OF $\pi/2$ --- VA'S AT $x = \frac{k\pi}{2}$ FOR ANY ODD INTEGER k

2. Determine the locations of two consecutive asymptotes of the graph of $y = \cot(5x + \pi)$

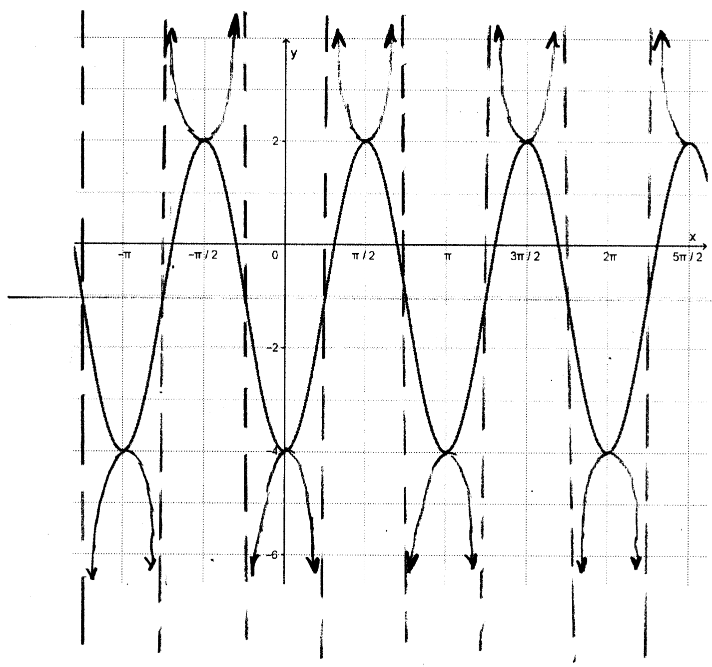
$y = \cot x$ HAS VA'S WHEN $\tan x = 0$

$\tan x = 0 \Rightarrow \underbrace{x = 0 \text{ or } x = \pi}_{2 \text{ CONSECUTIVE}}$

$5x + \pi = 0 \Rightarrow x = -\frac{\pi}{5}$

$5x + \pi = \pi \Rightarrow x = 0$

3. Shown below is the graph of $y = -1 + 3\cos(2x - \pi)$. Use the given graph to sketch the graph of $y = -1 + 3\sec(2x - \pi)$.



ASYPTOTES AT THE LOCATIONS WHERE $\cos(2x - \pi) = 0$

OR

$-1 + 3\cos(2x - \pi) = -1$

$y = -1$

ONCE THE ASYMPTOTES ARE SKETCHED, THE REST IS EASY.

4. Sketch a careful and detailed graph of $y = 3\cot \pi x$. Include two full periods. Label your axes.

SEE ATTACHED SHEET.

5. Sketch a careful and detailed graph of $y = 5\csc(x - \frac{\pi}{6})$. Include two full periods. Label your axes.

SEE ATTACHED SHEET.

6. Use your knowledge of the values of the trigonometric functions at special angles to determine the exact value of each of the following. Do not use a calculator.

(a) $\arcsin 0 = \boxed{0}$

BECAUSE $\sin(0) = 0$

(b) $\tan^{-1} 1 = \boxed{\frac{\pi}{4}}$ BECAUSE $\tan \frac{\pi}{4} = 1$

(c) $\arccos\left(-\frac{1}{2}\right) = \boxed{\frac{2\pi}{3}}$ BECAUSE $\cos \frac{2\pi}{3} = -\frac{1}{2}$

(d) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{\pi}{4}}$ BECAUSE $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

(e) $\cos^{-1} 1 = \boxed{0}$ BECAUSE $\cos 0 = 1$.

(f) $\arccos 2$ $\boxed{\text{NOT DEFINED}}$ BECAUSE NO REAL NUMBER HAS A COSINE OF 2.

(g) $\sin^{-1} \frac{\sqrt{3}}{2} = \boxed{\frac{\pi}{3}}$ BECAUSE $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

BE SURE TO USE THE CORRECT BRANCH.

7. Because the tangent and arctangent functions are inverses, Fernanda thought that it must be true that $\tan(\tan^{-1}(17)) = 17$. Is she correct? Explain why or why not.

YES, 17 IS IN THE DOMAIN OF THE INVERSE TANGENT FUNCTION, SO THE TANGENT WILL UNDO ITS ACTION.

8. Determine the exact value of $\tan^{-1}(\tan 2\pi)$.

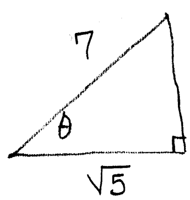
2π IS NOT IN THE RESTRICTED DOMAIN (BRANCH) OF THE TANGENT FUNCTION.

REWRITE $\tan^{-1}(\tan 2\pi) = \tan^{-1}(\tan 0)$

$= \boxed{0.}$

9. Use a right triangle to find the exact value of $\sin(\underbrace{\cos^{-1}\sqrt{5}/7}_{\theta})$.

$\cos\theta = \frac{\sqrt{5}}{7}$



$\sqrt{49-5} = \sqrt{44}$

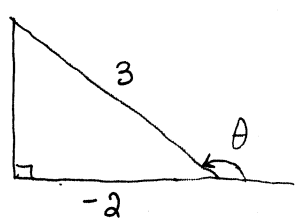
$= 2\sqrt{11}$

$\sin\theta = \frac{2\sqrt{11}}{7}$

10. Use a right triangle to find the exact value of $\tan[\underbrace{\cos^{-1}(-2/3)}_{\theta}]$.

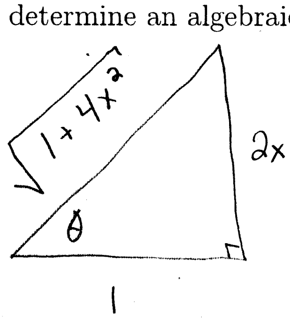
$\sqrt{9-4}$

$= \sqrt{5}$



$\tan\theta = -\frac{\sqrt{5}}{2}$

11. Use a right triangle to determine an algebraic expression for $\sin[\underbrace{\tan^{-1}(2x)}_{\theta}]$.



$\sin\theta = \frac{2x}{\sqrt{1+4x^2}}$

12. Use a calculator to determine the value of each of the following. Write you answers in radian measure rounded to the nearest tenth.

(a) $\sec^{-1} 4.5$ $= X$

$\sec^{-1} 4.5 = \cos^{-1}\left(\frac{1}{4.5}\right) \approx \boxed{1.3}$

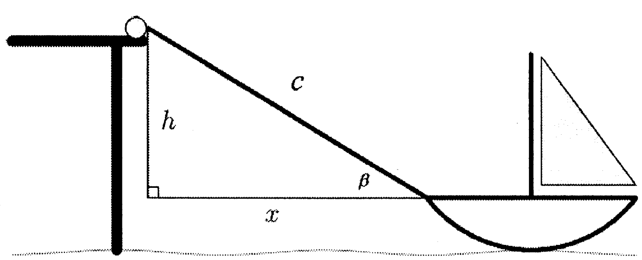
IF $\sec X = 4.5$

IF $\cos X = \frac{1}{4.5}$

(b) $\arctan 17$ $\approx \boxed{1.5}$

(c) $\cos^{-1} 0.25$ $\approx \boxed{1.3}$

13. A boat is being pulled toward a dock as shown in the figure below. For each part of this problem, use the given information to determine the angle β. Give your answer in degree measure, rounded to the nearest tenth of a degree.



(a) $h = 2\,\text{m}$ and $x = 7\,\text{m}$

$\tan\beta = \frac{h}{x} = \frac{2}{7} \Rightarrow \beta = \tan^{-1}\left(\frac{2}{7}\right) \approx \boxed{15.9^\circ}$

(b) $c = 13\,\text{ft}$ and $x = 12\,\text{ft}$

$\cos\beta = \frac{x}{c} = \frac{12}{13} \Rightarrow \beta = \cos^{-1}\left(\frac{12}{13}\right) \approx \boxed{22.6^\circ}$

14. True or false?

(a) $\tan^2 x = \tan x^2$

FALSE $\quad \tan^2 x = (\tan x)(\tan x)$

$\neq \tan(x \cdot x)$

(b) $\sin(-x) = \sin x$

FALSE. $\quad \sin(-x) = -\sin x$

(c) $\cos(-x) = \cos x$

TRUE

(d) $\cos\left(\theta - \frac{\pi}{2}\right) = \sin\theta$ TRUE; THINK ABOUT THE GRAPH OF EACH SIDE.

(e) $\sin^2 x + \cos^2 x = \sec^2 x - \tan^2 x$ TRUE, $1 = 1$

15. Simplify each expression.

(a) $\sin x - \csc^2 x \sin x$

$$\sin x\,(1 - \csc^2 x) = \sin x\,(-\cot^2 x) = -\frac{\cos^2 x}{\sin x} = -\cos x \cot x$$

(b) $\sin x \cos^2 x - \sin x$

$$\sin x\,(\cos^2 x - 1) = (\sin x)(-\sin^2 x) = \boxed{-\sin^3 x}$$

16. Factor each expression. THINK ABOUT $4x^2 + x - 3$.

(a) $4\tan^2 u + \tan u - 3$

$(4\tan u - 3)(\tan u + 1)$

(b) $2\csc^2 w - 7\csc w + 6$ THINK ABOUT $2x^2 - 7x + 6$

$(2\csc w - 3)(\csc w - 2)$

17. Rewrite and factor: $\csc^2 x - \cot x - 3$

$$1 + \cot^2 x - \cot x - 3$$

$$\cot^2 x - \cot x - 2$$

$$(\cot x - 2)(\cot x + 1)$$

18. Add and simplify: $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

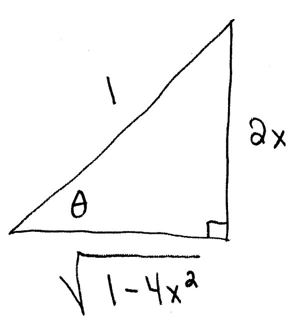
$$\frac{\sin\theta\,(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)} + \frac{\cos\theta}{\sin\theta}$$

$$\frac{\cancel{\sin\theta}\,(1-\cos\theta)}{\underbrace{1-\cos^2\theta}_{\sin^{\cancel{2}}\theta}} + \frac{\cos\theta}{\sin\theta} = \frac{1-\cos\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} = \csc\theta$$

19. Show that $\cos(\underbrace{\sin^{-1} 2x}_{\theta}) = \sqrt{1-4x^2}$.



$$\cos\theta = \frac{\sqrt{1-4x^2}}{1}$$

or

$$\cos\theta = \sqrt{1-4x^2}$$

✓

20. Verify each identity.

(a) $\dfrac{\sec^2\beta - 1}{\sec^2\beta} = \sin^2\beta$

$$\frac{\sec^2\beta}{\sec^2\beta} - \frac{1}{\sec^2\beta} = 1 - \frac{1}{\sec^2\beta} = 1 - \cos^2\beta = \sin^2\beta \checkmark$$

(b) $(\tan^2 x + 1)(\cos^2 x - 1) = -\tan^2 x$

$$\underbrace{\tan^2 x \cos^2 x} + \cos^2 x - \tan^2 x - 1$$

$$\underbrace{\sin^2 x + \cos^2 x} - \tan^2 x - 1$$

$$1 - \tan^2 x - 1 = -\tan^2 x \checkmark$$

(c) $\tan\alpha + \cot\alpha = \sec\alpha\csc\alpha$

$$\frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha} = \frac{\sin\alpha\sin\alpha}{\cos\alpha\sin\alpha} + \frac{\cos\alpha\cos\alpha}{\sin\alpha\cos\alpha}$$

$$= \frac{\sin^2\alpha + \cos^2\alpha}{\cos\alpha\sin\alpha} = \frac{1}{\cos\alpha\sin\alpha} = \frac{1}{\cos\alpha}\ \frac{1}{\sin\alpha}$$

$$= \sec\alpha\csc\alpha \checkmark$$

(d) $\sec u + \tan u = \dfrac{\cos u}{1 - \sin u}$

$$\frac{\cos u(1+\sin u)}{(1-\sin u)(1+\sin u)} = \frac{\cos u(1+\sin u)}{1-\sin^2 u} = \frac{\cancel{\cos u}(1+\sin u)}{\cos^{\cancel{2}} u}$$

$$= \frac{1+\sin u}{\cos u} = \frac{1}{\cos u} + \frac{\sin u}{\cos u} = \sec u + \tan u \checkmark$$

21. Find the exact solutions: $3\tan^2 x - 1 = 0$
(Find all solutions.)

$$\tan^2 x = \frac{1}{3}$$

$$\tan x = \pm\sqrt{\frac{1}{3}} = \pm\frac{1}{\sqrt{3}}$$

$$\tan x = \frac{1}{\sqrt{3}} \qquad \tan x = -\frac{1}{\sqrt{3}}$$

$$x = \frac{\pi}{6} \qquad x = -\frac{\pi}{6}$$

ALL SOLUTIONS...

$x = \frac{\pi}{6} + k\pi$, $x = -\frac{\pi}{6} + k\pi$, WHERE k IS ANY INTEGER.

22. Find the exact solutions: $2\sin^2 x + 3\cos x - 3 = 0$
(Find all solutions.)

$$2(1-\cos^2 x) + 3\cos x - 3 = 0$$

$$2 - 2\cos^2 x + 3\cos x - 3 = 0$$

$$2\cos^2 x - 3\cos x + 1 = 0$$

$$(2\cos x - 1)(\cos x - 1) = 0$$

$$\cos x = \frac{1}{2} \quad \text{OR} \quad \cos x = 1$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3} \quad \text{OR} \quad x = 0$$

ALL SOLUTIONS...

$x = \frac{\pi}{3} + 2k\pi$, $\frac{5\pi}{3} + 2k\pi$, $2k\pi$ FOR ANY INTEGER k

23. Find the exact solutions: $2\sin 4t - \sqrt{3} = 0$
(Find all solutions.)

LET $u = 4t$

$2\sin u = \sqrt{3}$

$\sin u = \frac{\sqrt{3}}{2} \Rightarrow u = \frac{\pi}{3}, \frac{2\pi}{3}$

ALL SOLUTIONS IN u...

$u = \frac{\pi}{3} + 2k\pi, \quad \frac{2\pi}{3} + 2k\pi$

RESUB TO GET ALL SOLUTIONS IN t...

$t = \frac{\pi}{12} + \frac{k\pi}{2}, \quad \frac{\pi}{6} + \frac{k\pi}{2}$ FOR ANY INTEGER k

24. Find the exact solutions in the interval $[0, 2\pi)$: $\sin^2 x + 2\sin x - 1 = 0$

$u^2 + 2u - 1 = 0$

USE QF... $u = \frac{-2 \pm \sqrt{4 - 4(1)(-1)}}{2} = \frac{-2 \pm \sqrt{8}}{2}$

$= \frac{-2 \pm 2\sqrt{2}}{2}$

$= -1 \pm \sqrt{2}$

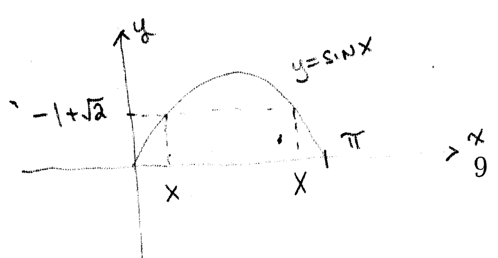
$\sin x = -1 - \sqrt{2}$

NOT POSSIBLE.

$\sin x = -1 + \sqrt{2}$

$x = \sin^{-1}(-1 + \sqrt{2}) \approx 0.427$

$x = \pi - \sin^{-1}(-1 + \sqrt{2}) \approx 2.715$



#4

START WITH $y = 3\tan \pi x$

ASYMPTOTES ...

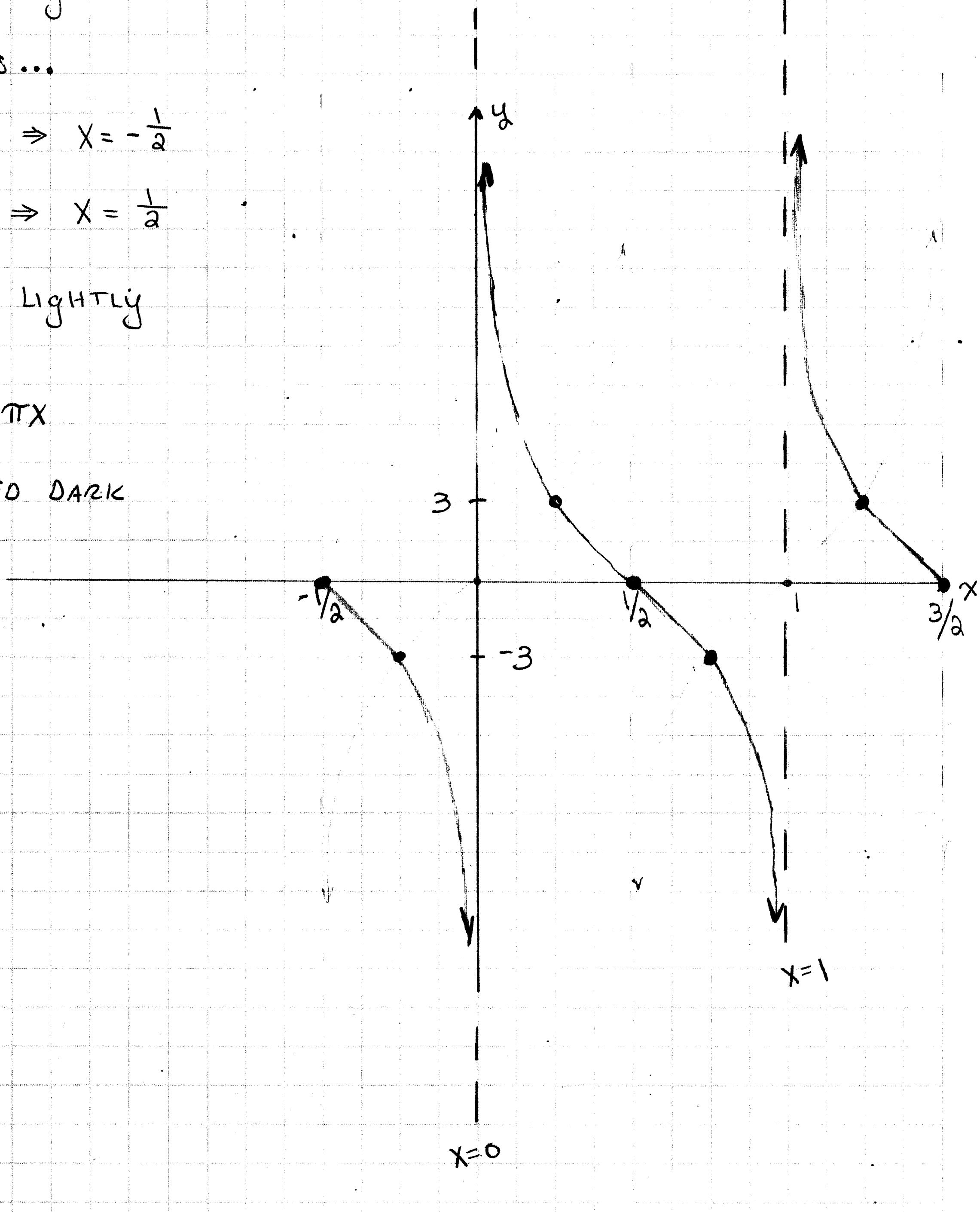
$\pi x = -\frac{\pi}{2} \Rightarrow x = -\frac{1}{2}$

$\pi x = \frac{\pi}{2} \Rightarrow x = \frac{1}{2}$

GRAPHED LIGHTLY

$y = 3\cot \pi x$

GRAPHED DARK



#5

START WITH GRAPH OF

$y = 5\sin\left(x - \frac{\pi}{6}\right)$

GRAPHED LIGHTLY

Amp = 5

Period = 2π

Shift = $\frac{\pi}{6}$ right

$y = 5\csc\left(x - \frac{\pi}{6}\right)$

GRAPHED DARK