Math 130 - Review 3
November 3, 2019

Name Key

Score ________

These problems may help you review for Test 3. Your actual test will not be as long as this review packet. For other practice problems, refer to the suggested homework problems. When providing exact answers, simplify as much as possible. For each triangle described below, a is opposite α, b is opposite β, and c is opposite γ (unless otherwise indicated).

1. Write 165° as the sum or difference of two of our familiar unit-circle angles. Then use the appropriate sum or difference formula(s) to find the exact values of the sine, cosine, and tangent of 165°. Do not use a calculator for this problem.

$165° = 120° + 45°$

$\sin(120° + 45°) = \sin 120° \cos 45° + \sin 45° \cos 120°$
$= \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(-\frac{1}{2}\right) = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

$\cos(120° + 45°) = \cos 120° \cos 45° - \sin 120° \sin 45°$
$= \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} = \boxed{\frac{-\sqrt{2}-\sqrt{6}}{4}}$

$\tan 165° = \frac{\sin 165°}{\cos 165°} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{-(\sqrt{2}+\sqrt{6})}}$

2. Repeat the problem above for 255°.

$255° = 300° - 45°$

$\sin(300° - 45°) = \sin 300° \cos 45° - \sin 45° \cos 300°$
$= -\frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) = \boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$

$\cos(300° - 45°) = \cos 300° \cos 45° + \sin 300° \sin 45°$
$= \frac{1}{2}\frac{\sqrt{2}}{2} + \left(\frac{-\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

$\tan 255° = \frac{\sin 255°}{\cos 255°} = \boxed{\frac{-(\sqrt{6}+\sqrt{2})}{\sqrt{2}-\sqrt{6}}}$

3. Find the exact value of $\sin\frac{\pi}{12}\cos\frac{\pi}{4} + \cos\frac{\pi}{12}\sin\frac{\pi}{4}$. Do not use a calculator for this problem.

$= \sin\left(\frac{\pi}{12} + \frac{\pi}{4}\right) = \sin\left(\frac{4\pi}{12}\right)$

$= \sin\left(\frac{\pi}{3}\right) = \boxed{\frac{\sqrt{3}}{2}}$

$\cos^2 u = 1 - \frac{49}{625}$

$= \frac{576}{625}$

$\underbrace{\cos u = -24/25}$

$\sin^2 v = 1 - \frac{16}{25}$

$= \frac{9}{25}$

$\underbrace{\sin v = -\frac{3}{5}}$

4. Suppose u and v are 3rd quadrant angles with $\sin u = -7/25$ and $\cos v = -4/5$. Find the exact values of $\cos(u - v)$, $\sin(u - v)$, and $\tan(u - v)$. Do not use a calculator for this problem.

$$\cos(u-v) = \cos u \cos v + \sin u \sin v = -\frac{24}{25}\left(-\frac{4}{5}\right) + \left(-\frac{7}{25}\right)\left(-\frac{3}{5}\right)$$

$$= \frac{96+21}{125} = \boxed{\frac{117}{125}}$$

$$\sin(u-v) = \sin u \cos v - \sin v \cos u$$

$$= \left(-\frac{7}{25}\right)\left(-\frac{4}{5}\right) - \left(-\frac{3}{5}\right)\left(-\frac{24}{25}\right)$$

$$= \frac{28-72}{125} = -\frac{44}{125}$$

$$\tan(u-v) = -\frac{44}{117}$$

5. Write $\cot(\theta - \pi)$ as a function of only θ. Do not use a calculator for this problem.

$$\tan(\theta - \pi) = \frac{\tan\theta - \tan\pi}{1 + \tan\theta \tan\pi} = \frac{\tan\theta - 0}{1} = \tan\theta$$

$$\cot(\theta - \pi) = \frac{1}{\tan(\theta-\pi)} = \frac{1}{\tan\theta} = \boxed{\cot(\theta)}$$

6. Find all solutions. Do not use a calculator.

(a) $\sin(x + \pi) - \sin x + 1 = 0$

$$\sin x \underbrace{\cancel{\cos\pi}}_{-1} + \underbrace{\cancel{\sin\pi}}_{0} \cos x - \sin x + 1 = 0$$

$$-\sin x - \sin x + 1 = 0$$

$$\sin x = \frac{1}{2} \Rightarrow \boxed{x = \frac{\pi}{6}, \frac{5\pi}{6} + 2k\pi}$$

(b) $\cos\left(x + \frac{\pi}{4}\right) - \cos\left(x - \frac{\pi}{4}\right) = 1$

$$\cancel{\cos x \cos\frac{\pi}{4}} - \sin x \sin\frac{\pi}{4} - \left(\cancel{\cos x \cos\frac{\pi}{4}} + \sin x \sin\frac{\pi}{4}\right) = 1$$

$$-2(\sin x)\left(\frac{\sqrt{2}}{2}\right) = 1$$

$$\sin x = -\frac{1}{\sqrt{2}} \Rightarrow \boxed{x = \frac{5\pi}{4}, \frac{7\pi}{4} + 2k\pi}$$

$$\sin^2\beta = 1 - \frac{16}{25} = \frac{9}{25}$$

$$\sin\beta = \frac{3}{5}$$

7. Given that β is a 2nd quadrant angle with $\cos\beta = -4/5$, find the exact values of $\sin 2\beta$, $\cos 2\beta$, and $\tan 2\beta$. Do not use a calculator for this problem.

$$\sin 2\beta = 2\sin\beta\cos\beta = 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right) = -\frac{24}{25}$$

$$\cos 2\beta = \cos^2\beta - \sin^2\beta = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$$\tan 2\beta = \frac{\sin 2\beta}{\cos 2\beta} = -\frac{24}{7}$$

8. Given that u is a 1st quadrant angle with $\cos u = 7/25$, find the exact value of $\sin(u/2)$, $\cos(u/2)$, and $\tan(u/2)$. Do not use a calculator for this problem.

$\frac{u}{2}$ IS A 1ST QUAD ANGLE

$$\sin\frac{u}{2} = \sqrt{\frac{1 - 7/25}{2}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$\cos\frac{u}{2} = \sqrt{\frac{1 + 7/25}{2}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$\tan\frac{u}{2} = \frac{\sin(u/2)}{\cos(u/2)} = \frac{3}{4}$$

9. Use a product-to-sum formula to rewrite $7\cos(-5x)\sin 3x$.

$$7\cos(-5x)\sin 3x = \frac{7}{2}\left[\sin(3x-5x) + \sin(3x+5x)\right]$$

$$= \frac{7}{2}\left[\sin(-2x) + \sin 8x\right]$$

$$= \frac{7}{2}\left[\sin 8x - \sin 2x\right]$$

10. Use a sum-to-product formula to rewrite $\cos x + \cos 4x$.

$$\cos x + \cos 4x = 2\cos\left(\frac{5}{2}x\right)\cos\left(-\frac{3}{2}x\right)$$

$$= 2\cos\left(\frac{5}{2}x\right)\cos\left(\frac{3}{2}x\right)$$

DOUBLE ANGLE 11. Find all solutions. Do not use a calculator.

(a) $\sin 2x - \sin x = 0$

$2\sin x \cos x - \sin x = 0$

$\sin x\,(2\cos x - 1) = 0$

$\sin x = 0$ OR $\cos x = \frac{1}{2}$

$\boxed{x = 0, \pi + 2k\pi}$ $\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3} + 2k\pi}$

SUM-TO-PRODUCT (b) $\underbrace{\sin 6x + \sin 2x}$ $= 0$

$2\sin 4x \cdot \cos 2x = 0$

$\sin 4x = 0$ OR $\cos 2x = 0$

$4x = 0, \pi + 2k\pi$ $2x = \frac{\pi}{2}, \frac{3\pi}{2} + 2k\pi$

$\boxed{x = 0, \frac{\pi}{4} + \frac{1}{2}k\pi}$ $\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4} + k\pi}$

DOUBLE ANGLE (c) $\sin\frac{x}{2} + \cos x = 0$

$u = \frac{x}{2}$

$\sin u + \cos 2u = 0$

$\sin u + 1 - 2\sin^2 u = 0$

$2\sin^2 u - \sin u - 1 = 0$

$(2\sin u + 1)(\sin u - 1) = 0$

$\sin u = -\frac{1}{2}$ $\sin u = 1$

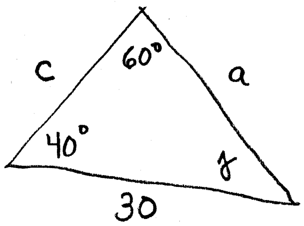
$u = \frac{7\pi}{6}, \frac{11\pi}{6} + 2k\pi$ $u = \frac{\pi}{2} + 2k\pi$

$u = x/2$

$$\boxed{\begin{aligned} x &= \frac{7\pi}{3}, \frac{11\pi}{3} + 4k\pi \\ x &= \pi + 4k\pi \end{aligned}}$$

12. Solve each triangle. Round to the nearest hundredth.

(a) $\alpha = 40°$, $\beta = 60°$, $b = 30$

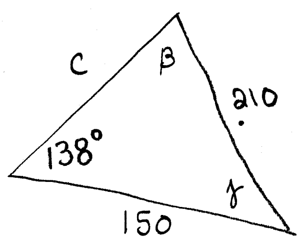


$$\frac{\sin 60°}{30} = \frac{\sin 40°}{a} \Rightarrow a = 22.27$$

$$\gamma = 180° - (40° + 60°) \Rightarrow \gamma = 80°$$

$$\frac{\sin 60°}{30} = \frac{\sin 80°}{c} \Rightarrow c = 34.11$$

(b) $\alpha = 138°$, $a = 210$, $b = 150$

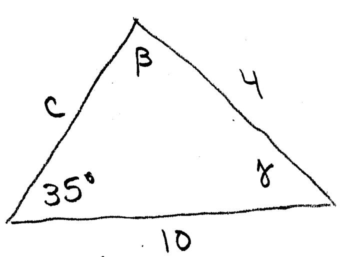


$$\frac{\sin 138°}{210} = \frac{\sin \beta}{150} \Rightarrow \beta = 28.55° \text{ or } \cancel{151.45°}$$

$$\gamma = 180° - (138° + 28.55°) \Rightarrow \gamma = 13.45°$$

$$\frac{\sin 138°}{210} = \frac{\sin 13.45°}{c} \Rightarrow c = 73.00$$

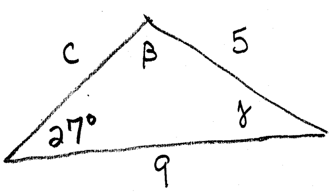
(c) $\alpha = 35°$, $a = 4$, $b = 10$



$$\frac{\sin 35°}{4} = \frac{\sin \beta}{10} \Rightarrow \underbrace{\sin \beta = 1.43}$$

No such β. $\Rightarrow$ No such $\triangle$

(d) $\alpha = 27°$, $a = 5$, $c = 9$



$$\frac{\sin 27°}{5} = \frac{\sin \beta}{9} \Rightarrow \beta = 54.80° \text{ or } 125.20°$$

$\beta = 54.80°$

$$\gamma = 180° - (27° + 54.80°) \Rightarrow \gamma = 98.20°$$

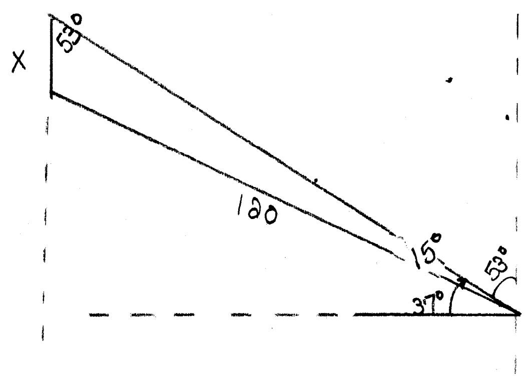
$$\frac{\sin 27°}{5} = \frac{\sin 98.20°}{c} \Rightarrow c = 10.90$$

$\beta = 125.20°$

$$\gamma = 180° - (27° + 125.20°) \Rightarrow \gamma = 27.80°$$

$$\frac{\sin 27°}{5} = \frac{\sin 27.80°}{c} \Rightarrow c = 5.14$$

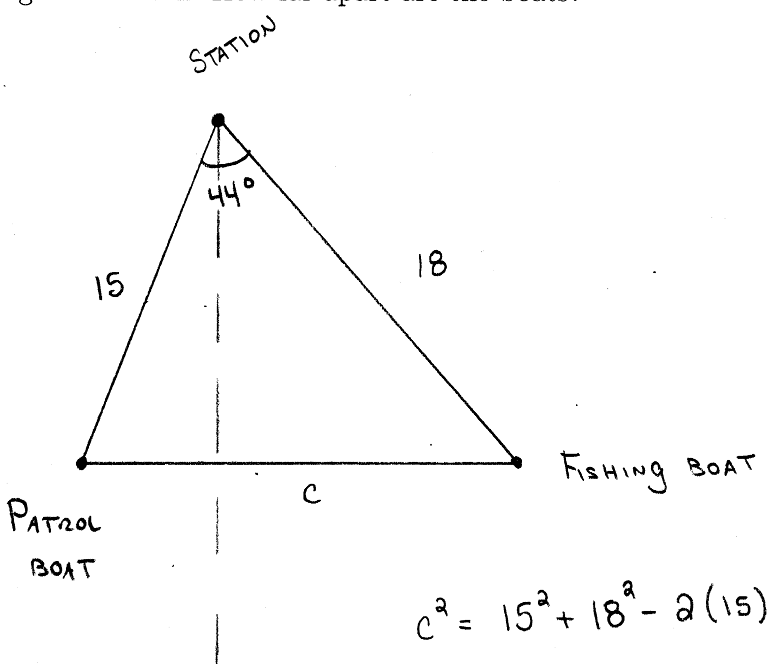
13. In a hilly area, a distance of 120 feet was measured down the slope of a hill from the base of a tree. From this point, the angles of elevation to the top and base of the tree are 37° and 22°, respectively. How tall is the tree?



$$\frac{\sin 53^\circ}{120} = \frac{\sin 15^\circ}{X}$$

$$\boxed{X = 38.9 \text{ FT}}$$

14. A fishing boat adrift at sea indicated its positions as 18 miles S 36° E from a coast guard station. A coast guard patrol boat indicated its position at 15 miles S 8° W of the coast guard station. How far apart are the boats?



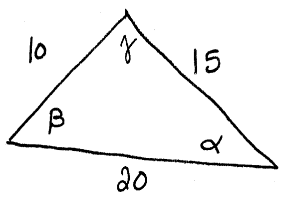
$$c^2 = 15^2 + 18^2 - 2(15)(18)\cos 44^\circ$$

$$= 160.5565\ldots$$

$$\Rightarrow \boxed{c \approx 12.7 \text{ MILES}}$$

15. Solve each triangle. Round to the nearest hundredth.

(a) $a = 10$, $b = 15$, $c = 20$

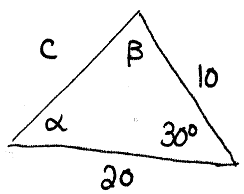


$$20^2 = 10^2 + 15^2 - 2(10)(15)\cos\gamma \Rightarrow \gamma = 104.48^\circ$$

$$15^2 = 10^2 + 20^2 - 2(10)(20)\cos\beta \Rightarrow \beta = 46.57^\circ$$

$$10^2 = 15^2 + 20^2 - 2(15)(20)\cos\alpha \Rightarrow \alpha = 28.96^\circ$$

(b) $a = 10$, $b = 20$, $\gamma = 30°$



$$c^2 = 10^2 + 20^2 - 2(10)(20)\cos 30^\circ \Rightarrow c = 12.39$$

$$\frac{\sin 30^\circ}{12.39} = \frac{\sin\beta}{20} \Rightarrow \cancel{\beta = 53.81^\circ} \text{ or } \beta = 126.19^\circ$$

$$\cancel{\alpha = 96.19^\circ} \text{ or } \alpha = 23.81^\circ$$

No way. Short side opp. small $\angle$

16. Carry out the indicated operation. Write your result in standard form.

(a) $(5 + 3i) + (2 - i)^2$

$$5 + 3i + 4 - 2i - 2i + i^2 = 8 - i$$

(b) $i^5 - i(3 - 6i) = i - 3i + 6i^2 = -6 - 2i$

(c) $i(3 - 7i)(2 + 4i) = i(6 + 12i - 14i - 28i^2) = i(34 - 2i)$

$$= 2 + 34i$$

(d) $\dfrac{8 + 7i}{2i - 3}$

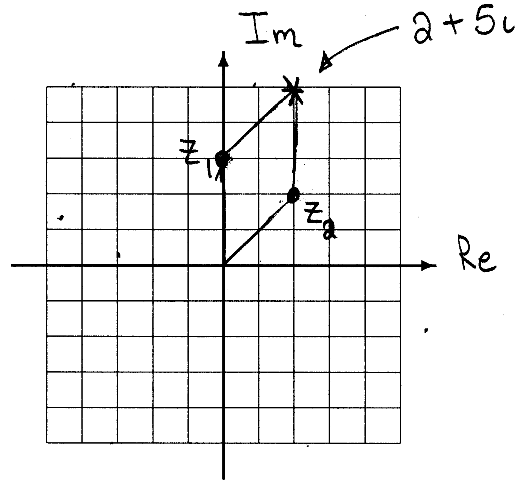
$$\frac{8+7i}{-3+2i} \cdot \frac{-3-2i}{-3-2i} = \frac{-24-16i-21i-14i^2}{9-4i^2}$$

$$= \frac{-10-37i}{13} = -\frac{10}{13} - \frac{37}{13}i$$

17. Plot, compute, and illustrate in the complex plane.

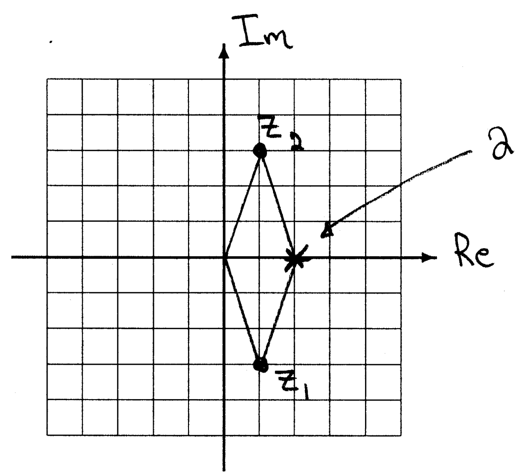
(a) $3i + (2 + 2i)$

z_1 z_2



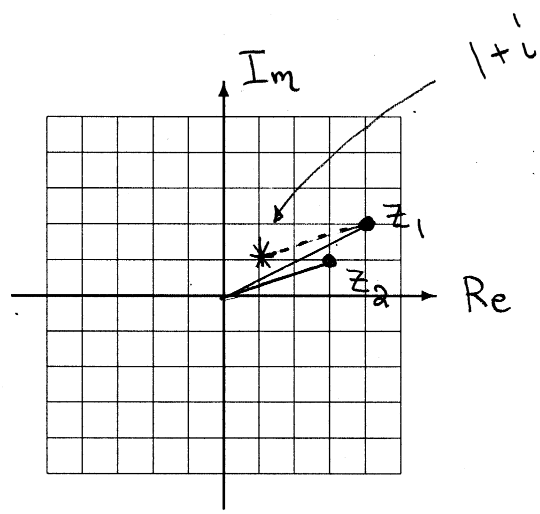
(b) $(1 - 3i) + (1 + 3i)$

z_1 z_2



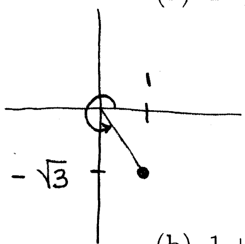
(c) $(4 + 2i) - (3 + i)$

z_1 z_2



18. Write in polar (trigonometric) form.

(a) $1 - \sqrt{3}\,i$

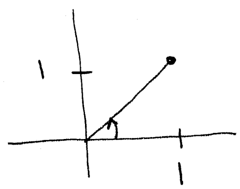


$r = \sqrt{1+3} = 2.$

$\tan^{-1} \frac{-\sqrt{3}}{1} = -60^\circ \Rightarrow \theta = 300^\circ$

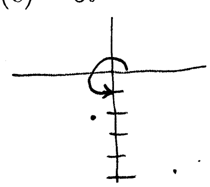
$2(\cos 300^\circ + i \sin 300^\circ)$

(b) $1 + i$



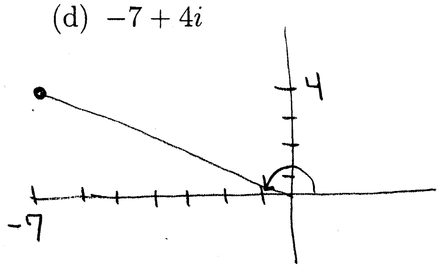
$\sqrt{2}\,(\cos 45^\circ + i \sin 45^\circ)$

(c) $-5i$



$5(\cos 270^\circ + i \sin 270^\circ)$

(d) $-7 + 4i$



$r = \sqrt{(-7)^2 + 4^2} = \sqrt{49+16} = \sqrt{65}$

$\tan^{-1} \frac{4}{-7} = -29.74^\circ \Rightarrow \theta = 150.26^\circ$

$\sqrt{65}\,(\cos 150.26^\circ + i \sin 150.26^\circ)$

19. Find the product $z_1 z_2$ in polar (trigonometric) form.

(a) $z_1 = \frac{5}{3}(\cos 120^\circ + i \sin 120^\circ), \qquad z_2 = \frac{2}{3}(\cos 30^\circ + i \sin 30^\circ)$

$\frac{5}{3} \cdot \frac{2}{3} \left[\cos 150^\circ + i \sin 150^\circ \right] = \frac{10}{9}(\cos 150^\circ + i \sin 150^\circ)$

(b) $z_1 = \frac{1}{2}(\cos 100^\circ + i \sin 100^\circ), \qquad z_2 = \frac{4}{5}(\cos 300^\circ + i \sin 300^\circ)$

$\frac{1}{2} \cdot \frac{4}{5} \left[\cos 400^\circ + i \sin 400^\circ \right] = \frac{2}{5}(\cos 40^\circ + i \sin 40^\circ)$

20. Find the quotient z_1/z_2 in polar (trigonometric) form.

(a) $z_1 = 12(\cos 92^\circ + i \sin 92^\circ)$, $\quad z_2 = 2(\cos 122^\circ + i\sin 122^\circ)$

$$\frac{12}{2}\left(\cos(-30^\circ) + i\sin(-30^\circ)\right)$$

$$= \boxed{6(\cos 330^\circ + i\sin 330^\circ)}$$

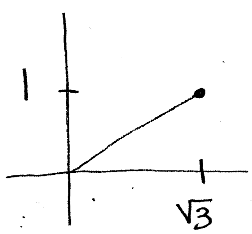
(b) $z_1 = 2(\cos\pi + i\sin\pi)$, $\quad z_2 = 3[\cos(\pi/3) + i\sin(\pi/3)]$

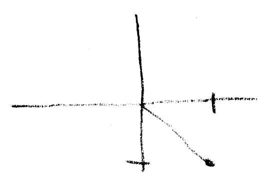
$$\pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$\boxed{\frac{2}{3}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)}$$

21. Write the complex numbers in polar form. Then find their product and quotient.

$$z_1 = \sqrt{3} + i, \qquad z_2 = 1 - i$$





$z_1 \ldots$

$$\tan^{-1}\frac{1}{\sqrt{3}} = 30^\circ$$

$$\Rightarrow \theta = 30^\circ$$

$$r_1 = \sqrt{3+1} = 2$$

$$z_1 = 2(\cos 30^\circ + i\sin 30^\circ)$$

$z_2 \ldots$

$$r_2 = \sqrt{1+1} = \sqrt{2}$$

$$\theta = 315^\circ$$

$$z_2 = \sqrt{2}(\cos 315^\circ + i\sin 315^\circ)$$

$$z_1 z_2 = \boxed{2\sqrt{2}(\cos 345^\circ + i\sin 345^\circ)}$$

$$\frac{z_1}{z_2} = \sqrt{2}\left(\cos(-285^\circ) + i\sin(-285^\circ)\right)$$

$$= \boxed{\sqrt{2}(\cos 75^\circ + i\sin 75^\circ)}$$