Math 130 - Review 4

December 8, 2019

Name key

Score ________

These problems may help you review for the final exam. For other practice problems; refer to the suggested homework problems. When providing exact answers, simplify as much as possible. **To adequately prepare for the comprehensive final exam, you should also study the earlier review packets, as well as old tests.**

1. Let $z = 2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$. Use DeMoivre's theorem to compute z^3. Write your final answer in standard form.

$$z^3 = 2^3\left(\cos\frac{3\pi}{6} + i\sin\frac{3\pi}{6}\right)$$

$$= 8\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

$$= 8(0 + i) = 8i$$

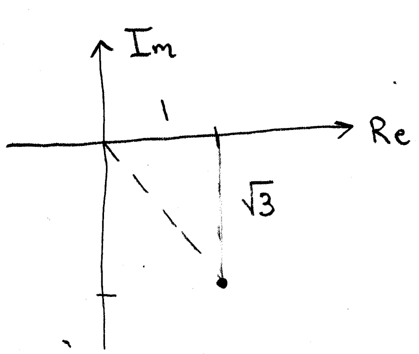
2. Let $z = 3(\cos 80° + i\sin 80°)$. Use DeMoivre's theorem to compute z^5. Write your final answer in standard form.

$$z^5 = 3^5(\cos 400° + i\sin 400°)$$

$$= 243(\cos 40° + i\sin 40°)$$

$$\approx 186.15 + 156.20i$$

3. Let $z = 1 - i\sqrt{3}$. Write z in polar form. Then use DeMoivre's theorem to compute z^2. Write your final answer in standard form.



$$r = \sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{4} = 2$$

$$\theta = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = -60° \text{ or } 300°$$

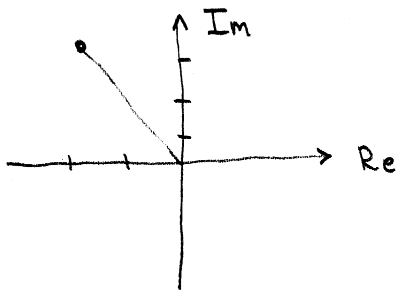
$$z = 2(\cos 300° + i\sin 300°)$$

$$z^2 = 4(\cos 600° + i\sin 600°)$$

$$= 4(\cos 240° + i\sin 240°)$$

$$= -2 - 2\sqrt{3}i$$

4. Let $z = -2 + 3i$. Write z in polar form. Then use DeMoivre's theorem to compute z^4. Write your final answer in standard form.



$r = \sqrt{4+9} = \sqrt{13}$

$\tan\theta = \frac{-3}{2}$ AND θ IS IN 2ND $\Rightarrow$ $\theta \approx 123.69°$

$z^4 = (\sqrt{13})^4 (\cos 4\theta + i \sin 4\theta)$

$= -119 + 120i$

5. Let $z = 81(\cos 80° + i \sin 80°)$. Compute the four 4th roots of z. Leave your answers in polar form.

$\sqrt[4]{81} = 3$

$\frac{80°}{4} = 20°$, $\frac{360°}{4} = 90°$

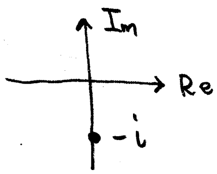
$z_1 = 3(\cos 20° + i \sin 20°)$

$z_2 = 3(\cos 110° + i \sin 110°)$

$z_3 = 3(\cos 200° + i \sin 200°)$

$z_4 = 3(\cos 290° + i \sin 290°)$

6. Write $z = -i$ in polar form. Then compute the 3 cube roots of z. Write your final answer in standard form.



$z = \cos 270° + i \sin 270°$

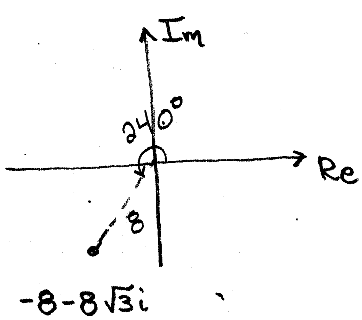
$\frac{270°}{3} = 90°$, $\frac{360°}{3} = 120°$

$z_1 = \cos 90° + i \sin 90°$
$= i$

$z_2 = \cos 210° + i \sin 210°$
$= -\frac{\sqrt{3}}{2} - \frac{1}{2}i$

$z_3 = \cos 330° + i \sin 330°$
$= \frac{\sqrt{3}}{2} - \frac{1}{2}i$

7. Let $z = -8 - 8\sqrt{3}i$. Write z in polar form. Then compute the four 4th roots of z. Leave your answers in polar form.



$z = 8(\cos 240° + i \sin 240°)$

$\frac{240°}{4} = 60°$, $\frac{360°}{4} = 90°$

$z_1 = \sqrt[4]{8}(\cos 60° + i \sin 60°)$

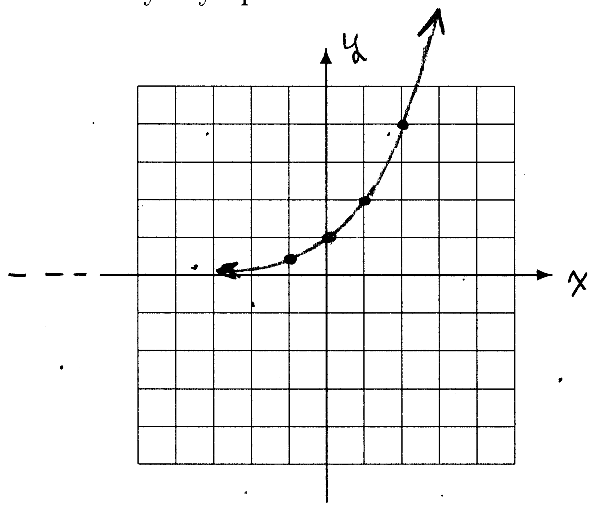
$z_2 = \sqrt[4]{8}(\cos 150° + i \sin 150°)$

$z_3 = \sqrt[4]{8}(\cos 240° + i \sin 240°)$

$z_4 = \sqrt[4]{8}(\cos 330° + i \sin 330°)$

8. Determine four points on the graph of $f(x) = 2^x$. Then plot your points and carefully sketch the graph. Indicate any asymptotes.

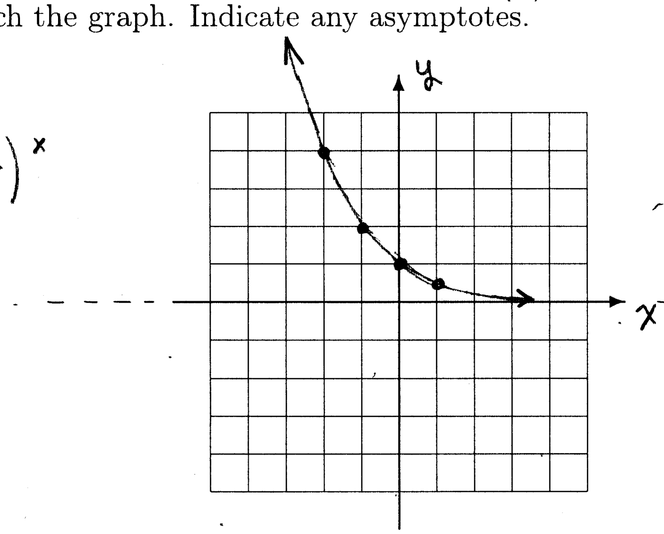
x	$y = 2^x$
0	1
-1	$1/2$
1	2
2	4



H.A. $y = 0$

9. Determine four points on the graph of $g(x) = \left(\frac{1}{2}\right)^x$. Then plot your points and carefully sketch the graph. Indicate any asymptotes.

x	$y = \left(\frac{1}{2}\right)^x$
0	1
-1	2
-2	4
1	$1/2$



H.A. $y = 0$

10. Determine the horizontal asymptote of the graph of $y = e^{x-2} - 3$.

$y = e^{x}$ & $y = e^{x-2}$

HAVE H.A. $y = 0$

$y = e^{x-2} - 3$

HAS H.A. $y = -3$

11. Determine the horizontal asymptote of the graph of $y = \left(\frac{2}{7}\right)^{x+4} + 9$.

$y = \left(\frac{2}{7}\right)^{x}$ & $y = \left(\frac{2}{7}\right)^{x+4}$

HAVE H.A. $y = 0$

$y = \left(\frac{2}{7}\right)^{x+4} + 9$

HAS H.A. $y = 9$

12. Determine the y-intercept of the graph of $y = 5^{x+1} + 8$.

$x = 0 \Rightarrow y = 5 + 8 = 13$

$(0, 13)$

13. Rewrite as an exponential equation: $\log_7 49 = 2$

$7^2 = 49$

14. Rewrite as an exponential equation: $\log_{1/2} 64 = -6$

$\left(\frac{1}{2}\right)^{-6} = 64$

15. Rewrite as a logarithmic equation: $3^6 = 729$

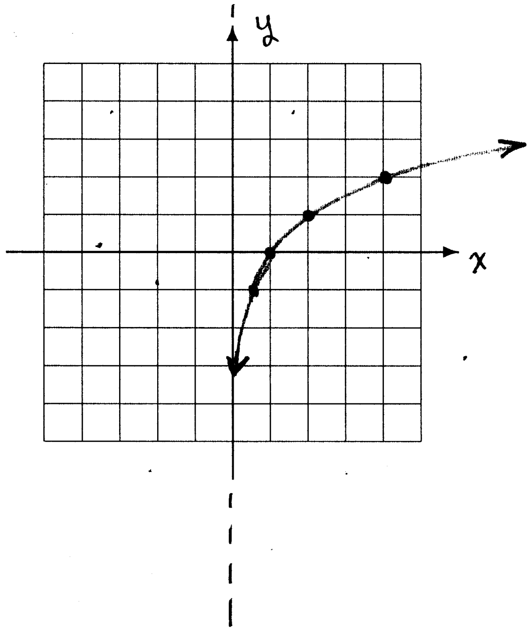
$\log_3 729 = 6$

16. Rewrite as a logarithmic equation: $2^{-5} = \frac{1}{32}$

$\log_2 \left(\frac{1}{32}\right) = -5$

17. Determine four points on the graph of $f(x) = \log_2 x$. Then plot your points and carefully sketch the graph. Indicate any asymptotes.

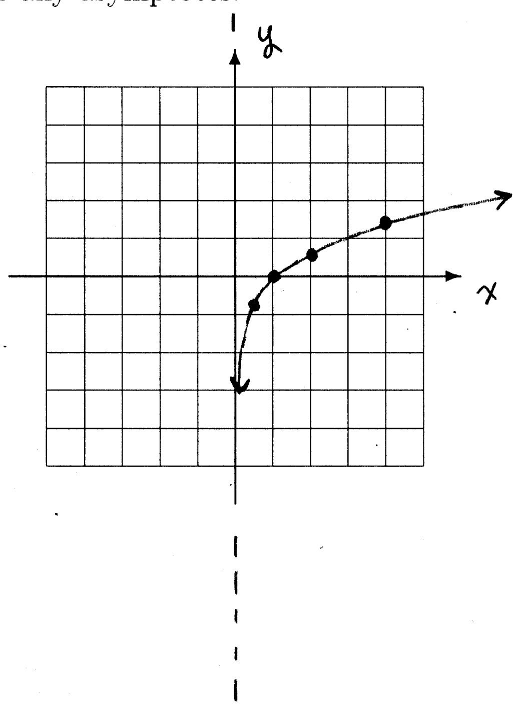
x	$y = \log_2 x$
$\frac{1}{2}$	-1
1	0
2	1
4	2



V.A. $x = 0$

18. Determine four points on the graph of $g(x) = \ln x$. Then plot your points and carefully sketch the graph. Indicate any asymptotes.

x	$y = \ln x$
$\frac{1}{2}$	-0.693
1	0
2	0.693
4	1.386



V.A. $x = 0$.

19. Explain how the graph of $y = 3 + \ln(x - 4)$ can be obtained from the graph of $y = \ln x$.

SHIFT THE GRAPH OF $y = \ln x$

FOUR UNITS RIGHT & THREE UNITS UP.

20. Determine the vertical asymptote of the graph of $y = \ln(x - 3)$.

$y = \ln x$ HAS V.A. $x = 0$

$y = \ln(x-3)$ HAS V.A. $x = 3$

21. Determine the vertical asymptote of the graph of $y = 4 + \log_5(x + 2)$.

V.A. IS $x = -2$

22. Determine the x-intercept of the graph of $y = \log_2(x - 7)$.

$\log_2(x-7) = 0$

$\Rightarrow x - 7 = 1$

$\Rightarrow x = 8$

$(8, 0)$

23. Use properties of logarithms to expand: $\ln(xyz)$

$$\ln x + \ln y + \ln z$$

24. Use properties of logarithms to expand: $\log\left(\frac{u^4}{w^3}\right)$

$$4 \log u - 3 \cdot \log w$$

25. Use properties of logarithms to expand: $\log_2\left(\frac{a^2b^3}{c^4d^5}\right)$

$$2 \log_2 a + 3 \log_2 b - 4 \log_2 c - 5 \log_2 d$$

26. Write as a single logarithmic expression: $\ln x + \ln 5$ $= \ln 5 + \ln x$

$$\ln 5x$$

27. Write as a single logarithmic expression: $2 \log x + 5 \log y - 7 \log z$

$$\log\left(\frac{x^2 y^5}{z^7}\right)$$

28. Simplify: $\ln e^8 + \ln e^4$

$$= 8 \underbrace{\ln e}_{1} + 4 \underbrace{\ln e}_{1}$$

$$= 8 + 4 = \boxed{12}$$

29. Use the change-of-base formula to write $\log_5 6$ in terms of natural logarithms. Then use your calculator to compute the value. Round to the nearest hundredth.

$$\log_5 6 = \boxed{\frac{\ln 6}{\ln 5} \approx 1.11}$$

30. Use the change-of-base formula to write $\log_7 63$ in terms of common (base-10) logarithms. Then use your calculator to compute the value. Round to the nearest hundredth.

$$\log_7 63 = \boxed{\frac{\log 63}{\log 7} \approx 2.13}$$

31. Solve for x: $16 = 8^{x-3}$

$$2^4 = (2^3)^{x-3} \longrightarrow 4 = 3(x-3)$$

$$4 = 3x - 9$$

$$13 = 3x \qquad \boxed{x = \frac{13}{3}}$$

32. Solve for x: $\dfrac{4\log_2(x+12)}{4} = \dfrac{12}{4}$

$$\log_2(x+12) = 3$$

$$2^3 = x + 12 \longrightarrow 8 = x + 12$$

$$\boxed{x = -4}$$

33. Solve for x: $e^{x^2} = e^{3x+4}$

$$x^2 = 3x + 4$$

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$$\boxed{\begin{array}{l} x = 4 \\ x = -1 \end{array}}$$

34. Solve for x: $e^{2x} - 7e^x + 12 = 0$

$u = e^x$

$u^2 - 7u + 12 = 0$

$(u-3)(u-4) = 0$

$u = 3 \Rightarrow e^x = 3 \Rightarrow x = \ln 3$

$u = 4 \Rightarrow e^x = 4 \Rightarrow x = \ln 4$

35. Solve for x. Round your answer to the nearest hundredth. $2^{3x} = 15$

$\ln 2^{3x} = \ln 15$

$\dfrac{3x \ln 2}{3 \ln 2} = \dfrac{\ln 15}{3 \ln 2}$

$x \approx 1.30$

36. Solve for x. Round your answer to the nearest hundredth. $\dfrac{4\ln(x+6)}{4} = \dfrac{-8}{4}$

$\ln(x+6) = -2$

$x + 6 = e^{-2}$

$x = e^{-2} - 6 \approx -5.86$

37. Solve for x. Round your answer to the nearest hundredth. $\ln(x+3) - \ln 2 = 3$

$\ln\left(\dfrac{x+3}{2}\right) = 3$

$\dfrac{x+3}{2} = e^3$

$x + 3 = 2e^3$

$x = 2e^3 - 3$

≈ 37.17

38. Solve for x. Round your answer to the nearest hundredth. $\log_5(x-7) = 1 + \log_5(x+1)$

$\log_5(x-7) - \log_5(x+1) = 1$

$\log_5\left(\dfrac{x-7}{x+1}\right) = 1$

$\dfrac{x-7}{x+1} = 5$

$x - 7 = 5(x+1)$

$x - 7 = 5x + 5$

$-12 = 4x$

$x = -3$

BUT $x = -3$ CANNOT WORK.

NO SOLUTION

39. In an effort to control vegetation overgrowth, 100 rabbits are released into an isolated area that is free of predators. After one year, the rabbit population has increased to 500. Assuming exponential population growth, what will the population be after another 6 months?

LET'S USE

$$P(t) = P_0 e^{kt}$$

$P_0 = 100$ AND $P(1) = 500$.

$$500 = 100e^k$$
$$5 = e^k$$
$$\ln 5 = k$$

$$P(t) = 100e^{t \ln 5}$$

$$P(1.5) = 100e^{1.5 \ln 5}$$
$$\approx \boxed{1118 \text{ RABBITS}}$$

Plutonium-239 has a half-life of about 24,100 years. Use a model of the form $P(t) = Ae^{-kt}$ to determine the initial amount of Pu-239 if there were 0.4 grams remaining after 1000 years.

$$\text{HALF-LIFE} = 24100 \Rightarrow \frac{1}{2} = e^{24100k}$$
$$\ln \tfrac{1}{2} = 24100k$$
$$k = \frac{\ln(1/2)}{24100}$$

$$P(1000) = \underbrace{0.4 = P_0 e^{1000\left(\frac{\ln 1/2}{24100}\right)}}$$

$$P_0 = \frac{0.4}{e^{1000\left(\frac{\ln 1/2}{24100}\right)}} \approx \boxed{0.41167 \text{ grams}}$$

40. Polonium-210 has a half-life of 140 days. A scientist has 25 grams of Polonium-210. How many grams will remain after $\underbrace{\text{one year}}_{\text{365 DAYS}}$?

LET'S USE

$$P(t) = P_0 e^{kt}$$

$$P_0 = 25$$

$$\frac{1}{2} = e^{140k}$$

$$k = \frac{\ln(1/2)}{140}$$

$$P(t) = 25 e^{t \ln(1/2)/140}$$

$$\boxed{P(365) \approx 4.1 \text{ grams}}$$

41. One-hundred animals were released into a preserve where their population grows according to the model $P(t) = \dfrac{1000}{1 + 9e^{-0.1656t}}$, where t is measured in months. After how long will the population reach 750 animals?

$$\frac{1000}{1 + 9e^{-0.1656t}} = 750$$

$$\frac{1000}{750} = 1 + 9e^{-0.1656t}$$

$$\frac{\frac{1000}{7500} - 1}{9} = \frac{\cancel{9}e^{-0.1656t}}{\cancel{9}}$$

$$e^{-0.1656t} = \frac{\frac{1000}{750} - 1}{9}$$

$$-0.1656t = \ln\left(\frac{\frac{1000}{750} - 1}{9}\right)$$

$$\boxed{t \approx 19.9 \text{ MONTHS}}$$