Math 130 - Test 1

September 11, 2019

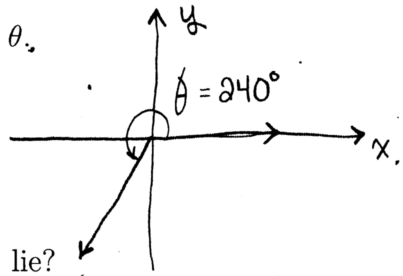
Name key

Score ________

Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) The angle θ lies in standard position and has measure $240°$.

   (a) Roughly sketch the angle θ.

   (b) In which quadrant does θ lie?

   3RD QUAD.

   (c) Determine the radian measure of θ. Write your answer in the form $\frac{m}{n}\pi$ with the fractional part in lowest terms.

   $$240° \cdot \frac{\pi}{180°} = \frac{24}{18}\pi = \boxed{\frac{4\pi}{3}}$$

   (d) Determine two (additional) coterminal angles, one positive and one negative. Write both answers in degree measure.

   $-120°$ & $600°$

2. (4 points) Determine the supplement of $8\pi/15$. ADD UP TO π OR $180°$

   (a) Write your answer in radian measure in the form $\frac{m}{n}\pi$.

   $$\pi - \frac{8\pi}{15} = \boxed{\frac{7\pi}{15}}$$

   (b) Write your answer in degree measure.

   $$\frac{7\pi}{15} \cdot \frac{180°}{\pi} = \frac{1260°}{15} = \boxed{84°}$$

3. (3 points) A 138° angle is swept out on a circle of radius 20 in. Determine the length of the arc. Round your answer to the nearest hundredth of an inch.

$$\text{ARC LENGTH} = \text{RADIAN MEASURE} \times \text{RADIUS}$$
$$= (138°)\left(\frac{\pi}{180°}\right) \times (20 \text{ in}) \approx \boxed{48.17 \text{ in}}$$

4. (7 points) Vinyl record albums typically have a 12 in diameter, and they play on a turntable at $33\frac{1}{3}$ revolutions per minute.

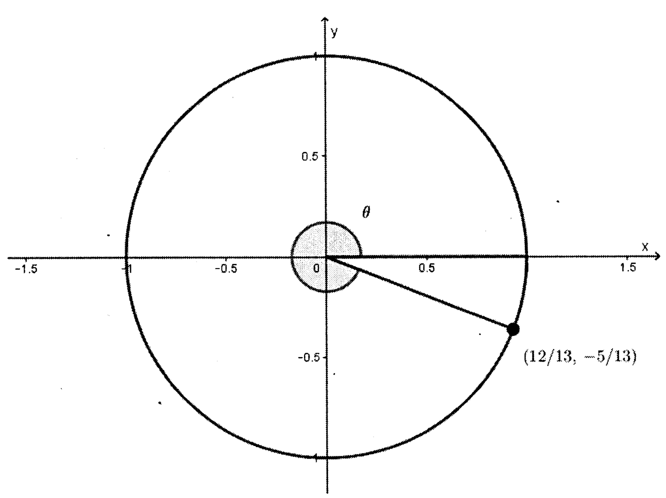
(a) Determine the angular speed of a record album. Give your answer in radians per minute.

$$\omega = \frac{\text{ANGLE}}{\text{TIME}} = \frac{(33\frac{1}{3})(2\pi)}{1 \text{ min}} = \frac{200\pi}{3} \text{ RADIANS}/\text{min}$$
$$\approx 209.44 \text{ RAD}/\text{min}$$

(b) Determine the linear speed of a point on the outer edge of a record album. Give your answer in inches per minute. Write in decimal form, rounded to the nearest tenth.

$$v = \omega \times \text{RADIUS} = \left(\frac{200\pi}{3}\right)(6) \text{ in}/\text{min} = 400\pi \text{ in}/\text{min}$$
$$\approx \boxed{1256.6 \text{ in}/\text{min}}$$

5. (6 points) Find the exact values of the six trigonometric functions at θ. Write your answers as fractions in lowest terms.



$\cos\theta = \frac{12}{13}$

$\sin\theta = -\frac{5}{13}$

$\tan\theta = -\frac{5}{12}$

$\sec\theta = \frac{13}{12}$

$\csc\theta = -\frac{13}{5}$

$\cot\theta = -\frac{12}{5}$

6. (6 points) Write the exact values of each of the following. Do not use your calculator.

(a) $\cos 60°$ $= \boxed{\frac{1}{2}}$

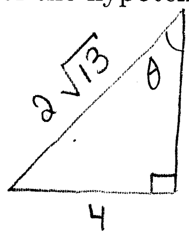
(b) $\sin(\pi/4)$ $\boxed{\frac{\sqrt{2}}{2}}$

(c) $\tan 30°$ $= \sin 30° / \cos 30° = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

(d) $\sin(\pi/6)$ $= \boxed{\frac{1}{2}}$

7. (8 points) The two legs of a right triangle have lengths 4 and 6.

(a) Determine the length of the hypotenuse.



$h^2 = 4^2 + 6^2$

$h^2 = 16 + 36 = 52$

$\boxed{h = \sqrt{52} = 2\sqrt{13}}$

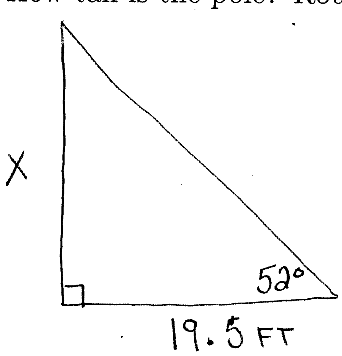
(b) Let θ be the smallest angle of the triangle. Determine the exact values of the six trigonometric functions at θ. You do not have to rationalize your denominators, but otherwise write your fractions as simple as possible.

$\sin\theta = \frac{4}{2\sqrt{13}} = \frac{2}{\sqrt{13}}$ $\quad$ $\csc\theta = \frac{\sqrt{13}}{2}$

$\cos\theta = \frac{6}{2\sqrt{13}} = \frac{3}{\sqrt{13}}$ $\quad$ $\sec\theta = \frac{\sqrt{13}}{3}$

$\tan\theta = \frac{4}{6} = \frac{2}{3}$ $\quad$ $\cot\theta = \frac{3}{2}$

8. (5 points) A guy wire runs from the ground to the top of a utility pole. The wire is attached to the ground 19.5 ft from the base of the pole, and the angle formed between the wire and the ground measures 52°. Assume that the pole is perpendicular to the ground. How tall is the pole? Round your answer to the nearest tenth of a foot.



$\tan 52° = \frac{X}{19.5}$

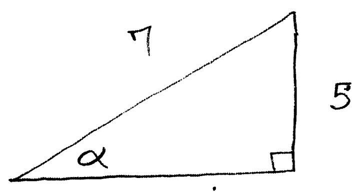
$X = 19.5 \tan 52°$

≈ 24.9589 FT

$\approx \boxed{25.0 \text{ FT}}$

$\sin \alpha = \frac{5}{7} = \frac{\text{OPP}}{\text{HYP}}$

9. (10 points) Sketch a right triangle with an acute angle α for which $\csc \alpha = \frac{7}{5}$. Then find the values of the other five trigonometric functions at α. You do not have to rationalize your denominators, but otherwise write your fractions as simple as possible.



$\sqrt{49-25} = \sqrt{24} = 2\sqrt{6}$

$\cos \alpha = \frac{2\sqrt{6}}{7}$ $\qquad \sec \alpha = \frac{7}{2\sqrt{6}}$

$\sin \alpha = \frac{5}{7}$ $\qquad \csc \alpha = \frac{7}{5}$

$\tan \alpha = \frac{5}{2\sqrt{6}}$ $\qquad \cot \alpha = \frac{2\sqrt{6}}{5}$

10. (6 points) For each part below, use the information to determine the quadrant in which θ lies.

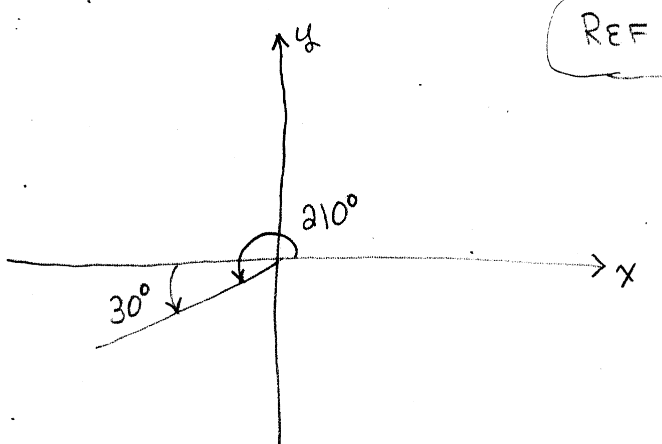
(a) $\csc \theta < 0$, $\sec \theta < 0$ $\Rightarrow$ SINE & COSINE ARE NEG $\Rightarrow$ $\boxed{\text{QUAD 3}}$

(b) $\sec \theta < 0$, $\cot \theta > 0$ $\Rightarrow$ COS NEG, TAN POS $\Rightarrow$ $\boxed{\text{QUAD 3}}$

(c) $\sec \theta > 0$, $\tan \theta > 0$ $\Rightarrow$ COS POS, TAN POS $\Rightarrow$ $\boxed{\text{QUAD 1}}$

11. (8 points) $\theta = 210°$. Determine the reference angle. Then, without using your calculator, determine the exact values of the six trigonometric functions at θ. Simplify your answers as much as possible.

$\boxed{\text{REF } \angle \text{ IS } 30°}$



$\cos \theta = -\cos 30° = -\frac{\sqrt{3}}{2}$ $\qquad \sec \theta = -\frac{2}{\sqrt{3}}$

$\sin \theta = -\sin 30° = -\frac{1}{2}$ $\qquad \csc \theta = -2$

$\tan \theta = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\qquad \cot \theta = \sqrt{3}$

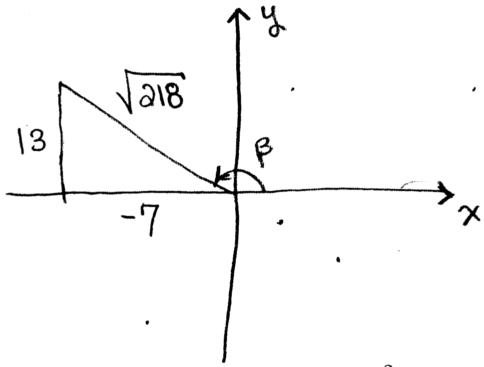
SINE NEG

COSINE NEG

2ND QUAD !

12. (6 points) $\overbrace{\tan\beta = -13/7 \text{ and } \sin\beta > 0}^{\text{2ND QUAD !}}$.
Find the exact values of $\sin\beta$ and $\cos\beta$. Simplify your answers as much as possible.

$$\sqrt{(13)^2+(7)^2} = \sqrt{218}$$



$$\boxed{\begin{aligned} \sin\beta &= \frac{13}{\sqrt{218}} = \frac{13\sqrt{218}}{218} \\ \cos\beta &= \frac{-7}{\sqrt{218}} = \frac{-7\sqrt{218}}{218} \end{aligned}}$$

13. (3 points) Starting with the identity $\sin^2\theta + \cos^2\theta = 1$, show how to obtain the new identity $1 + \cot^2\theta = \csc^2\theta$.

$$\sin^2\theta + \cos^2\theta = 1$$

DIVIDE BOTH SIDES BY $\sin^2\theta$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta} \Rightarrow 1 + \cot^2\theta = \csc^2\theta$$

14. (6 points) Use trig identities to transform one side of the equation into the other.

(a) $\underbrace{\cos\theta \tan\theta \csc\theta}$ $= 1$

$$\left(\frac{\cancel{\cos\theta}}{1}\right)\left(\frac{\cancel{\sin\theta}}{\cancel{\cos\theta}}\right)\left(\frac{1}{\cancel{\sin\theta}}\right) = 1 \quad ✓$$

(b) $\underbrace{\dfrac{\tan\alpha + \cot\alpha}{\tan\alpha}} = \csc^2\alpha$

$$\frac{\tan\alpha}{\tan\alpha} + \frac{\cot\alpha}{\tan\alpha}$$

$$1 + (\cot\alpha)(\cot\alpha)$$

$$1 + \cot^2\alpha = \csc^2\alpha \quad ✓$$

15. (5 points) For each equation, determine the amplitude and the period of the graph.

(a) $y = -5\sin(x+\pi)$

$\text{Amp} = |-5| = 5$ $\quad$ $\text{Period} = \frac{2\pi}{1} = 2\pi$

(b) $y = 2 + 5\cos(\frac{\pi}{2}x + 1)$

$\text{Amp} = 5$ $\quad$ $\text{Period} = \frac{2\pi}{\pi/2} = 4$

16. (4 points) Write an equation whose graph has the given characteristics: a sine curve with period π, an amplitude of 3, a left phase shift of $\pi/5$, and a vertical translation down 8 units.

$\frac{2\pi}{b} = \pi$

$\Downarrow$

$b = 2$

$y = -8 + 3\sin 2\left(x + \frac{\pi}{5}\right)$

OR

$y = -8 + 3\sin\left(2x + \frac{2\pi}{5}\right)$

17. (7 points) On the attached graph paper, sketch the graph of $y = 1 + 2\cos(x + \frac{\pi}{4})$. Label your graph well enough for a person to read it. (Include two full periods.)

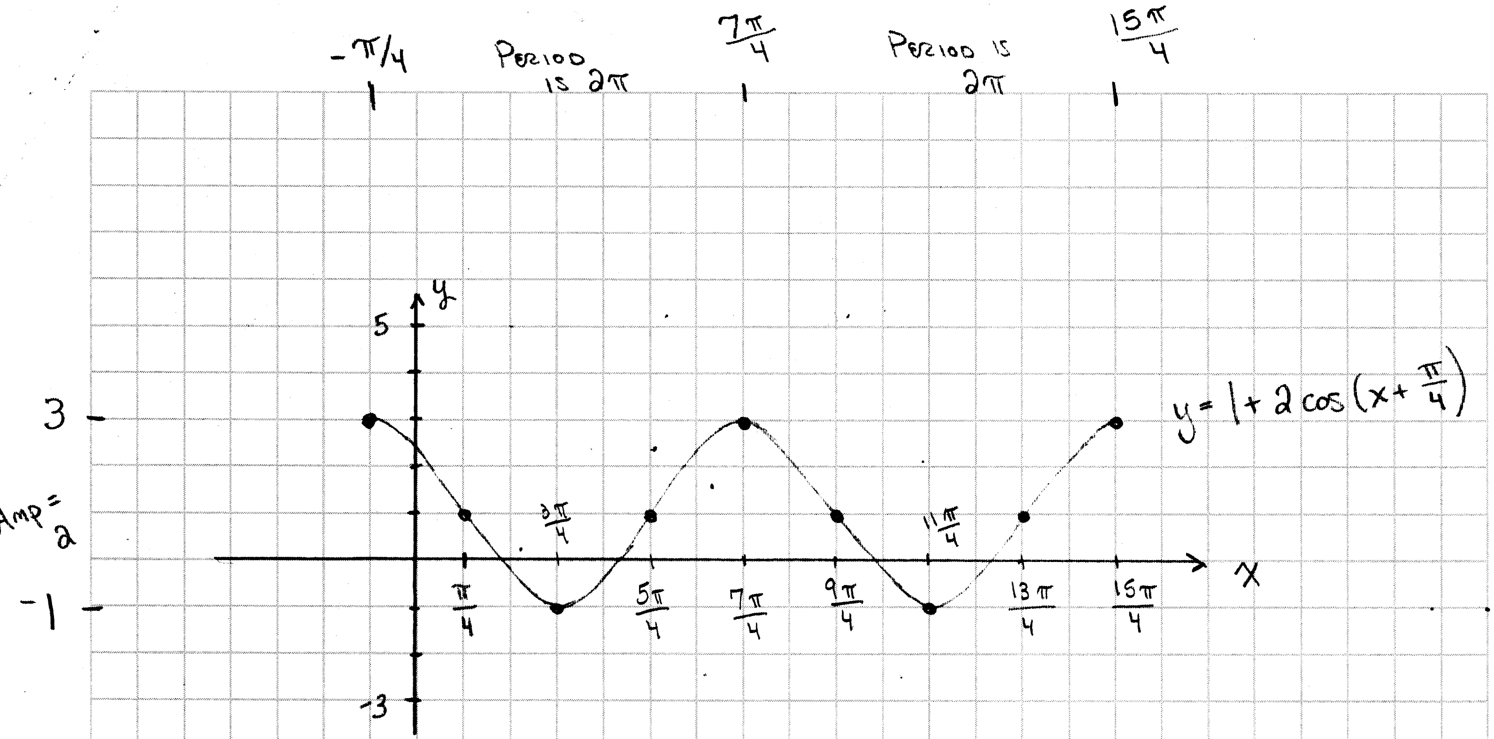
SEE ATTACHED.

Period 2π

Amp 2

Shifted 1 up

and $\frac{\pi}{4}$ left


-π/4
Period is 2π
7π/4
Period is 2π
15π/4
y
5
3
Amp = 2
-1
-3
x
π/4
2π/4
5π/4
7π/4
9π/4
11π/4
13π/4
15π/4
y = 1 + 2 cos(x + π/4)