Math 130 - Test 2
October 9, 2019

Name key

Score ________

Show all work to receive full credit. Supply explanations where necessary. When finding exact answers, simplify as much as possible. You may use your unit circle and trig identity card on any problem unless otherwise indicated.

1. (5 points) Determine the locations of two consecutive asymptotes of the graph of $y = 1 + 2\tan(3x + \frac{\pi}{4})$.

$3x + \frac{\pi}{4} = -\frac{\pi}{2}$

$3x = -\frac{3\pi}{4}$

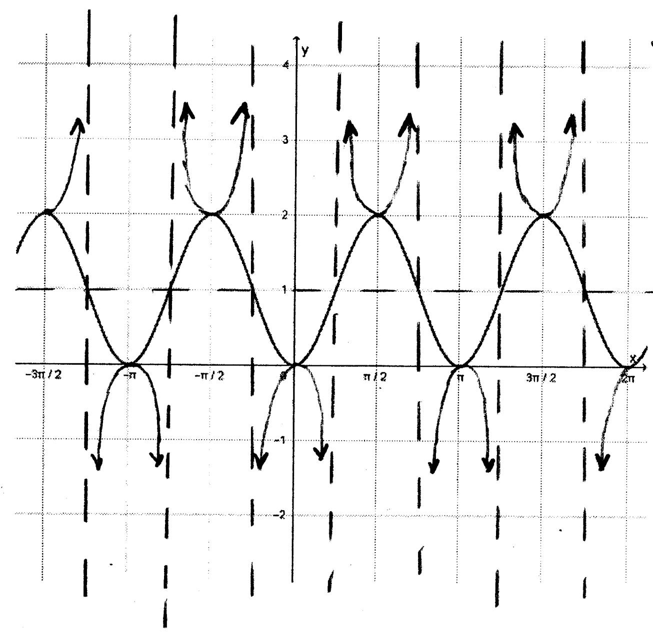
$\boxed{x = -\frac{\pi}{4}}$

$3x + \frac{\pi}{4} = \frac{\pi}{2}$

$3x = \frac{\pi}{4}$

$\boxed{x = \frac{\pi}{12}}$

2. (5 points) Shown below is the graph of $y = 1 - \sin(2x + \frac{\pi}{2})$. Use the given graph to sketch the graph of $y = 1 - \csc(2x + \frac{\pi}{2})$. Clearly indicate any special features of the graph such as asymptotes, intercepts, etc.



VERTICAL ASYMPTOTES AT LOCATIONS WHERE $\csc(2x + \frac{\pi}{2}) = 0$

y=1

3. (5 points) At which x-values does the graph of $y = \csc x$ have vertical asymptotes? Describe **all** such x-values.

GRAPH OF $y = \csc x$ HAS ASYMPTOTES AT EACH X-VALUE FOR WHICH $\sin x = 0$, THAT IS, AT EVERY INTEGER MULTIPLE OF π.

VA's: $x = k\pi$, k IS ANY INTEGER.

4. (8 points) On the attached graph paper, sketch a careful and detailed graph of $y = \frac{1}{2}\sec(x - \frac{\pi}{4})$. Include two full periods and be sure to label your axes.

SEE ATTACHED SHEET.

5. (10 points) Use your knowledge of the values of the trigonometric functions at special angles to determine the exact value of each of the following. Do not use a calculator.

(a) $\tan^{-1}\sqrt{3}$

$= \boxed{\frac{\pi}{3}}$ BECAUSE $\tan\frac{\pi}{3} = \frac{\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$.

(b) $\arccos\left(\frac{-\sqrt{3}}{2}\right) = \boxed{\frac{5\pi}{6}}$ BECAUSE $\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

(c) $\sin^{-1}\left(-\frac{1}{2}\right) = \boxed{-\frac{\pi}{6}}$ BECAUSE $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

(d) $\cos^{-1} 0 = \boxed{\frac{\pi}{2}}$ BECAUSE $\cos\frac{\pi}{2} = 0$

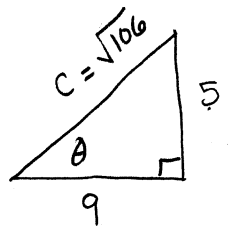
(e) $\arcsin\sqrt{5}$ $\boxed{\text{DOES NOT EXIST}}$

$\sqrt{5}$ IS TOO BIG TO BE THE SINE OF AN ANGLE.

6. (2 points) Determine the exact value of $\cos^{-1}(\cos 13\pi/6)$.

$= \cos^{-1}\left(\cos\frac{\pi}{6}\right) = \boxed{\frac{\pi}{6}}$

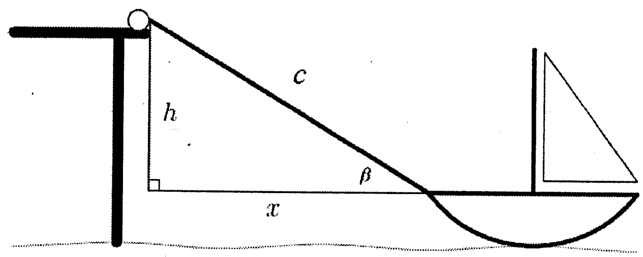
7. (5 points) Use a right triangle to find the exact value of $\cos(\underbrace{\tan^{-1}(5/9)}_{\theta})$.



$c^2 = 81 + 25 = 106$

$$\boxed{\cos\theta = \frac{9}{\sqrt{106}} = \frac{9\sqrt{106}}{106}}$$

8. (4 points) A boat is being pulled toward a dock as shown in the figure below. Determine the angle β if $h = 6\,\text{ft}$ and $c = 22\,\text{ft}$. Give your answer in degree measure, rounded to the nearest tenth of a degree.



$\sin\beta = \frac{6}{22}$

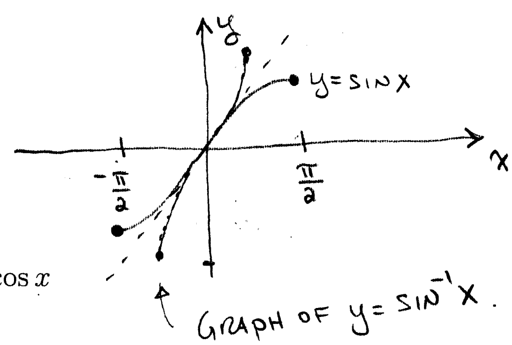
$\Downarrow$

$\beta = \sin^{-1}\left(\frac{6}{22}\right)$

$\approx \boxed{15.8°}$

9. (3 points) Explain how the graph of $y = \sin^{-1} x$ can be obtained from the graph of $y = \sin x$.

THE GRAPH OF $y = \sin^{-1} X$ IS THE GRAPH OF $y = \sin X,\ -\frac{\pi}{2} \le X \le \frac{\pi}{2}$, FLIPPED ABOUT THE LINE $y = X$



10. (3 points) Simplify the expression: $\cos x \sin^2 x - \cos x$

$\cos X\,(\sin^2 X - 1)$

$= \cos X\,(-\cos^2 X)$

$= \boxed{-\cos^3 X}$

11. (3 points) Rewrite and factor: $\sec^2 x + 5\tan x - 1$

$$\uparrow \tan^2 x + 1$$

$$\tan^2 x + 1 + 5\tan x - 1$$

$$= \tan^2 x + 5\tan x = \boxed{\tan x\,(\tan x + 5)}$$

12. (5 points) Rewrite $\dfrac{1}{1+\sin x}$ so that it is not in fractional form.

(Your final answer should contain only tangents and secants.)

$$\frac{1}{1+\sin x} \cdot \frac{1-\sin x}{1-\sin x} = \frac{1-\sin x}{1-\sin^2 x} = \frac{1-\sin x}{\cos^2 x} = \frac{1}{\cos^2 x} - \frac{\sin x}{\cos x}\left(\frac{1}{\cos x}\right)$$

$$= \boxed{\sec^2 x - \tan x \sec x}$$

13. (4 points) Verify the identity: $(\sec^2 -1)(\sin^2 x - 1) = -\sin^2 x$

$$(\sec^2 x - 1)(\sin^2 x - 1)$$

$$= \tan^2 x\,(\sin^2 x - 1)$$

$$= (\tan^2 x)(-\cos^2 x)$$

$$= \left(\frac{\sin^2 x}{\cancel{\cos^2 x}}\right)(-\cancel{\cos^2 x}) = (\sin^2 x)(-1) = -\sin^2 x$$

GOT IT!

14. (6 points) Verify the identity: $\csc\theta + \cot\theta = \dfrac{\sin\theta}{1-\cos\theta}$

$$\frac{\sin\theta}{1-\cos\theta} \cdot \frac{(1+\cos\theta)}{1+\cos\theta} = \frac{\sin\theta\,(1+\cos\theta)}{1-\cos^2\theta} = \frac{\cancel{\sin\theta}\,(1+\cos\theta)}{\sin^{\cancel{2}}\theta}$$

$$= \frac{1+\cos\theta}{\sin\theta}$$

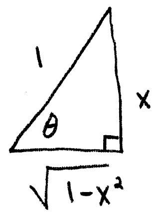
$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \csc\theta + \cot\theta$$

GOT IT!

15. (4 points) Use a right triangle to show that $\tan(\underbrace{\sin^{-1} x}_{\theta}) = \dfrac{x}{\sqrt{1-x^2}}$.

$\sin\theta = \dfrac{x}{1} \Rightarrow$

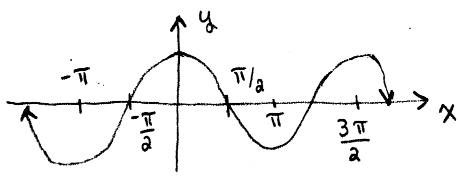


$\tan\theta = \dfrac{\text{OPP}}{\text{ADJ}} = \dfrac{x}{\sqrt{1-x^2}}$

GOT IT!

16. (8 points) Is this problem, you will establish the trig identity $\cos\left(x - \frac{\pi}{2}\right) = \sin x$.

(a) Draw a rough sketch of the graph of $y = \cos x$.



(b) Explain how the graph of $\cos\left(x - \frac{\pi}{2}\right)$ can be obtained from the graph of $y = \cos x$.

SHIFT THE GRAPH OF $y = \cos x$ IS SHIFTED $\frac{\pi}{2}$ UNITS RIGHT TO GET THE GRAPH OF $y = \cos\left(x - \frac{\pi}{2}\right)$.

(c) Thinking graphically, explain why it must be true that $\cos\left(x - \frac{\pi}{2}\right) = \sin x$.

IF YOU SHIFT THE GRAPH OF $y = \cos x$ $\frac{\pi}{2}$ UNITS RIGHT, YOU GET PRECISELY THE GRAPH OF $y = \sin x$.

17. (4 points) Find the exact solutions: $\underbrace{\cos x = \frac{1}{2}}$.
(Find all solutions.)

FROM UNIT CIRCLE, $x = \frac{\pi}{3}, \frac{5\pi}{3}$.

ALL SOLUTIONS ARE

$x = \frac{\pi}{3} + 2k\pi, \quad x = \frac{5\pi}{3} + 2k\pi$

WHERE k IS ANY INTEGER.

18. (6 points) Find the exact solutions: $4\sin^2 x - 3 = 0$
(Find all solutions.)

$$\sin^2 x = \frac{3}{4} \Rightarrow \underbrace{\sin x = \pm\frac{\sqrt{3}}{2}}$$

From unit circle

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

All solutions are

$$x = \frac{\pi}{3} + 2k\pi, \frac{2\pi}{3} + 2k\pi, \frac{4\pi}{3} + 2k\pi, \frac{5\pi}{3} + 2k\pi;$$

k is any integer

19. (6 points) Find the exact solutions in the interval $[0, 2\pi)$: $2\sin^2 x - 3\sin x + 1 = 0$

$$(2\sin x - 1)(\sin x - 1) = 0$$

$\sin x = \frac{1}{2}$ or $\sin x = 1$

$$x = \frac{\pi}{6}, \frac{5\pi}{6} \qquad x = \frac{\pi}{2}$$

20. (4 points) Adrianna correctly solved the equation $\sin x = \frac{1}{2}$. She found that $x = \frac{\pi}{6} + 2k\pi$ or $x = \frac{5\pi}{6} + 2k\pi$, where k is any integer. Use Adrianna's work to solve $\sin 5t = \frac{1}{2}$.

$$5t = \frac{\pi}{6} + 2k\pi$$
$$5t = \frac{5\pi}{6} + 2k\pi$$

$\Rightarrow$

$$t = \frac{\pi}{30} + \frac{2}{5}k\pi,$$
$$t = \frac{\pi}{6} + \frac{2}{5}k\pi,$$ where k is any integer

4 $\quad y = \frac{1}{2}\sec\left(x - \frac{\pi}{4}\right)$

Asymptotes at

$x = -\frac{\pi}{4} + k\pi$