Math 130 - Test 3
November 6, 2019

Name key

Score ________

Show all work to receive full credit. Supply explanations where necessary. When finding exact answers, simplify as much as possible. You may use your unit circle and trig identity card on any problem. For each triangle described below, a is opposite α, b is opposite β, and c is opposite γ (unless otherwise indicated).

1. (7 points [3,6]) Write 105° as the sum or difference of two of our familiar angles. Then use the appropriate sum or difference formula(s) to find the exact value of $\tan 105°$. Do not use a calculator for this problem.

$$\tan(105°) = \tan(45° + 60°)$$

$$= \frac{\tan 45° + \tan 60°}{1 - \tan 45° \tan 60°}$$

$$= \frac{1 + \sqrt{3}}{1 - (1)(\sqrt{3})} = \frac{1+\sqrt{3}}{1-\sqrt{3}} = -2 - \sqrt{3}$$

2. (8 points [3,6,9]) Use a sum formula to rewrite the equation. Then find the exact solutions in the interval $[0, 2\pi)$. Do not use a calculator for this problem.

$$\sin\left(x + \frac{\pi}{6}\right) - \frac{1}{2}\cos x = \frac{\sqrt{6}}{4}$$

$$\sin x \cos\frac{\pi}{6} + \cancel{\overbrace{\sin\frac{\pi}{6}}^{1/2} \cos x} - \cancel{\frac{1}{2}\cos x} = \frac{\sqrt{6}}{4}$$

$$(\sin x)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{6}}{4}$$

$$\sin x = \frac{\sqrt{2}}{2} \Rightarrow x = \frac{\pi}{4} \text{ or } \frac{3\pi}{4}$$

$$\cos^2\theta = 1 - \frac{9}{25} = \frac{16}{25}$$

$$\cos\theta = 4/5$$

3. (7 points [3,6]) Given that θ is a 4th quadrant angle with $\sin\theta = -3/5$, find the exact values of $\sin 2\theta$ and $\cos 2\theta$. Do not use a calculator for this problem.

$$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right) = -\frac{24}{25}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{4}{5}\right)^2 - \left(-\frac{3}{5}\right)^2 = \frac{16-9}{25} = \boxed{\frac{7}{25}}$$

4. (8 points [3,6,9]) Use a sum-to-product formula to rewrite the equation. Then find the exact solutions (find all solutions). Do not use a calculator for this problem.

$$\cos 2x - \cos 6x = 0$$

$$\cos 2x - \cos 6x = -2\sin 4x \sin(-2x)$$

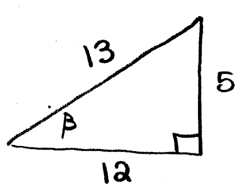
$$= 2\sin 4x \sin 2x = 0$$

$$\sin 4x = 0 \qquad \sin 2x = 0$$

$$4x = 0, \pi + 2k\pi \qquad 2x = 0, \pi + 2k\pi$$

$$x = 0, \frac{\pi}{4} + \frac{1}{2}k\pi \qquad x = 0, \frac{\pi}{2} + k\pi$$

5. (7 points [3,6]) Given that β is a 1st quadrant angle with $\tan\beta = 5/12$, find the exact value of $\sin\beta/2$. Do not use a calculator for this problem.



$$\cos\beta = \frac{12}{13}$$

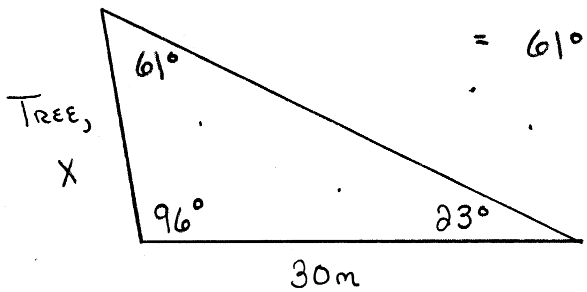
$$\sin\frac{\beta}{2} = \pm\sqrt{\frac{1-\cos\beta}{2}}$$

From Power Reducing Formula

$$\sin\frac{\beta}{2} = \sqrt{\frac{1-\cos\beta}{2}} = \sqrt{\frac{1-12/13}{2}} = \frac{1}{\sqrt{26}}$$

Pos. square root in Quad 1.

6. (7 points [7]) On flat ground, a tree leans away from an observer so that it makes a 96° angle with the ground. At a point 30 meters from the base of the tree, the angle of elevation to its top measures 23°. How tall is the tree?



$180° - (96° + 23°)$

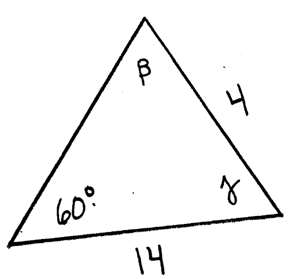
$= 61°$

$$\frac{\sin 61°}{30.} = \frac{\sin 23°}{X}$$

$$X = \frac{30 \sin 23°}{\sin 61°}$$

$$= \boxed{13.4 \text{ m}}$$

7. (5 points [7]) Show analytically (not with a picture) that there is no triangle for which $a = 4$ feet, $b = 14$ feet, and $\alpha = 60°$.



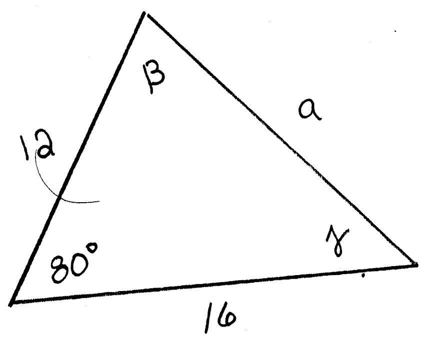
$$\frac{\sin 60°}{4} = \frac{\sin \beta}{14}$$

$$\underbrace{\sin \beta = \frac{14 \sin 60°}{4} = 3.0310\ldots}_{\text{No such angle } \beta}$$

$\Downarrow$

No such Δ.

8. (10 points [8]) Given the triangle with $\alpha = 80°$, $b = 16$ meters, and $c = 12$ meters, find the remaining angles and side length.



$a^2 = 12^2 + 16^2 - 2(12)(16)\cos 80°$

$a^2 = 333.3191 \Rightarrow \boxed{a = 18.26 \text{ m}}$

$12^2 = a^2 + 16^2 - 2(a)(16)\cos\gamma$

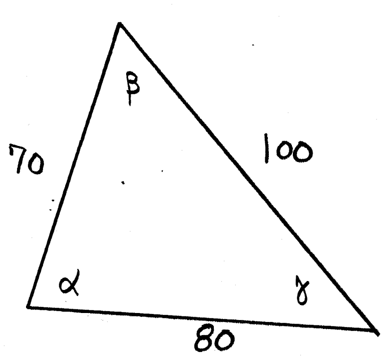
$\cos\gamma = 0.762115$

$\boxed{\gamma = 40.35°}$

$\beta = 180° - (80° + 40.35°) = 59.65°$

$\boxed{\beta = 59.65°}$

9. (8 points [8]) A triangular plot of land is bounded on each side by a busy street. The lengths of the sides of the plot of land are 70.00 m, 80.00 m, and 100.00 m. Determine the angles that the streets make with one another. Give your answers in degrees, rounded to the nearest hundredth.



$100^2 = 70^2 + 80^2 - 2(70)(80)\cos\alpha$

$\boxed{\alpha = 83.33°}$

$80^2 = 100^2 + 70^2 - 2(100)(70)\cos\beta$

$\boxed{\beta = 52.62°}$

$70^2 = 100^2 + 80^2 - 2(100)(80)\cos\gamma$

$\boxed{\gamma = 44.05°}$

10. (4 points [11]) Write $i(4-5i)^2$ as a complex number in standard form. Show all work.

$$i(4-5i)(4-5i) = i(16-20i-20i+25i^2)$$

$$= i(-9-40i) = -9i-40i^2 = \boxed{40-9i}$$

11. (3 points [11]) Determine the complex conjugate of each number.

(a) $-6-13i$ $\boxed{-6+13i}$

(b) πi $\boxed{-\pi i}$

(c) -17.25 $\boxed{-17.25}$

12. (4 points [11]) Write $\dfrac{5+i}{5-i}$ as a complex number in standard form. Show all work.

$$\frac{5+i}{5-i} \cdot \frac{5+i}{5+i} = \frac{25+10i+i^2}{25-i^2} = \frac{24+10i}{26} = \frac{12+5i}{13}$$

$$= \boxed{\frac{12}{13} + \frac{5}{13}i}$$

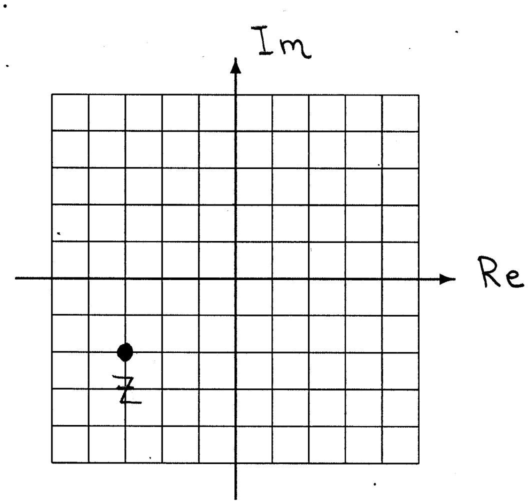
13. (4 points [11,12]) Plot the number $z = -3-2i$ in the complex plane (label your axes!). Then compute $|z|$.

$|z|$ = DISTANCE FROM ORIGIN

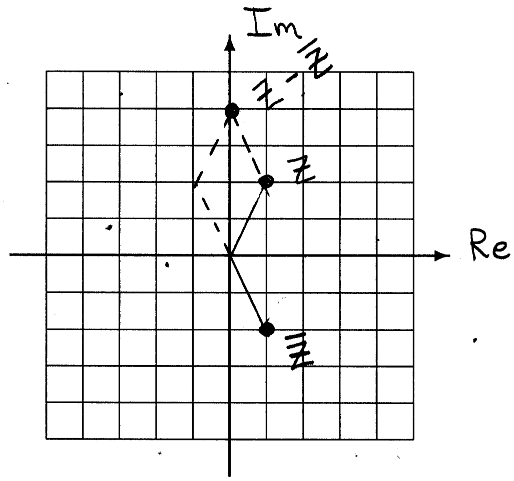
$$= \sqrt{(-3)^2+(-2)^2}$$

$$= \sqrt{9+4}$$

$$= \boxed{\sqrt{13}}$$

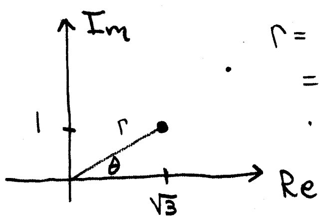


14. (4 points [11,12]) Let $z = 1+2i$. Plot z and $\bar{z}$ in the complex plane (label your axes!). Then use arrows to compute and illustrate $z - \bar{z}$. (Recall that $\bar{z}$ is the conjugate of z.)



$z - \bar{z} = \boxed{4i}$

15. (4 points [11]) Write the complex number $\sqrt{3}+i$ in polar (trigonometric) form. Show all work.



$r = |\sqrt{3}+i|$
$= \sqrt{3+1} = \sqrt{4}$
$= 2$

$\tan\theta = \dfrac{1}{\sqrt{3}} \Rightarrow \theta = 30°$

$\boxed{2(\cos 30° + i \sin 30°)}$

16. (6 points [11]) Let $z_1 = 3(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$ and $z_2 = 4(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$.

(a) Find the product $z_1 z_2$ in polar (trigonometric) form.

$\frac{\pi}{3} + \frac{\pi}{6} = \frac{3\pi}{6} = \frac{\pi}{2}$

$z_1 z_2 = (3)(4)\left[\cos\left(\frac{\pi}{3}+\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{3}+\frac{\pi}{6}\right)\right]$

$= \boxed{12\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)}$

(b) Write the product $z_1 z_2$ in standard form.

$12\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = 12(0+i)$

$= \boxed{12i}$

17. (4 points [11]) Let $z_1 = 9(\cos 50° + i\sin 50°)$ and $z_2 = 3(\cos 20° + i\sin 20°)$. Find the quotient z_1/z_2 in polar (trigonometric) form.

$\dfrac{z_1}{z_2} = \dfrac{9}{3}\left[\cos(50°-20°) + i\sin(50°-20°)\right]$

$= \boxed{3(\cos 30° + i\sin 30°)}$