Math 130 - Final Exam

December 11, 2019

Name key

Score ________

Show all work to receive full credit. Supply explanations where necessary. When finding exact answers, simplify as much as possible. You may use your unit circle and trig identity card on any problem. For each triangle described below, a is opposite α, b is opposite β, and c is opposite γ (unless otherwise indicated).

1. (3 points) Convert 285° to radian measure. Write your answer as a fraction of π in lowest terms.

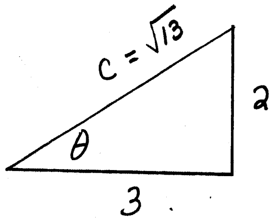
$$\frac{285°}{1} \cdot \frac{\pi}{180°} = \boxed{\frac{19\pi}{12}}$$

$285 = 15 \cdot 19$

$180 = 15 \cdot 12$

2. (10 points) Sketch a right triangle with an acute angle θ for which $\cot\theta = \frac{3}{2}$. Then find the values of the other five trigonometric functions at θ. You do not have to rationalize your denominators, but otherwise write your fractions as simple as possible.

$\cot\theta = \frac{3}{2} = \frac{\text{ADJ}}{\text{OPP}}$



$c = \sqrt{9+4}$

$= \sqrt{13}$

$\sin\theta = \frac{2}{\sqrt{13}}$

$\cos\theta = \frac{3}{\sqrt{13}}$

$\tan\theta = \frac{2}{3}$

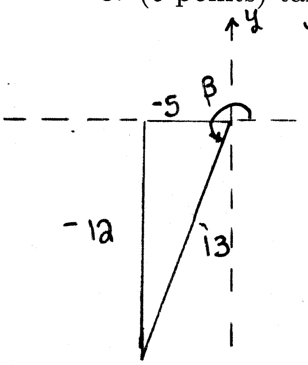
$\csc\theta = \frac{\sqrt{13}}{2}$

$\sec\theta = \frac{\sqrt{13}}{3}$

BOTH NEG IN QUAD 3

3. (6 points) $\tan\beta = 12/5$ and $\sin\beta < 0$. Find the exact values of $\sin\beta$ and $\cos\beta$.

β IN QUAD 3



$\sqrt{(-5)^2 + (-12)^2}$

$= \sqrt{169}$

$= 13$

$\sin\beta = \frac{-12}{13}$

$\cos\beta = \frac{-5}{13}$

4. (8 points) On the attached graph paper, sketch the graph of $y = 3\cos(2x + \pi)$. Label your graph well enough for a person to read it. Include two full periods.

SEE ATTACHED SHEET.

5. (10 points) Use your knowledge of the values of the trigonometric functions at special angles to determine the exact value of each of the following. Do not use a calculator. Show or explain how you got each answer.

(a) $\cos^{-1}\left(-\frac{1}{2}\right) = \boxed{\frac{2\pi}{3} = 120°}$ BECAUSE $\cos\frac{2\pi}{3} = -\frac{1}{2}$ AND $0 \le \frac{2\pi}{3} \le \pi$

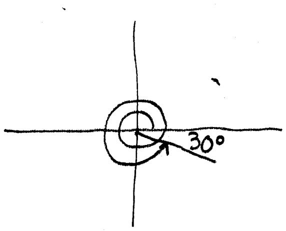
(b) $\tan^{-1} 1 = \boxed{\frac{\pi}{4} = 45°}$ BECAUSE $\tan\frac{\pi}{4} = 1$ AND $-\frac{\pi}{2} < \frac{\pi}{4} < \frac{\pi}{2}$

(c) $\arcsin\left(\frac{-1}{2}\right) = \boxed{-\frac{\pi}{6} = -30°}$ BECAUSE $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$ AND $-\frac{\pi}{2} \le -\frac{\pi}{6} \le \frac{\pi}{2}$

(d) $\sin^{-1} 1 = \boxed{\frac{\pi}{2} = 90°}$ BECAUSE $\sin\frac{\pi}{2} = 1$ AND $-\frac{\pi}{2} \le \frac{\pi}{2} \le \frac{\pi}{2}$

(e) $\arccos 2$ $\boxed{\text{DNE}}$ NO REAL ANGLE HAS A COSINE OF 2.

6. (4 points) Determine the exact value of $\sin^{-1}[\sin(23\pi/6)]$.



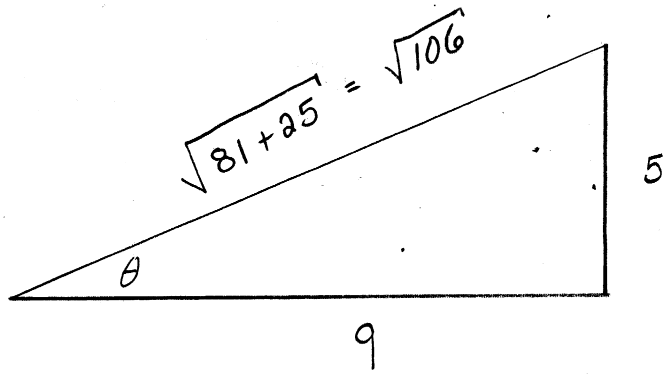
$\frac{23\pi}{6}$

$= \sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right) = \boxed{-\frac{\pi}{6}}$

$\theta = \tan^{-1}\left(\frac{5}{9}\right)$

7. (6 points) Use a right triangle to find the exact value of $\sin(\tan^{-1}(5/9))$.

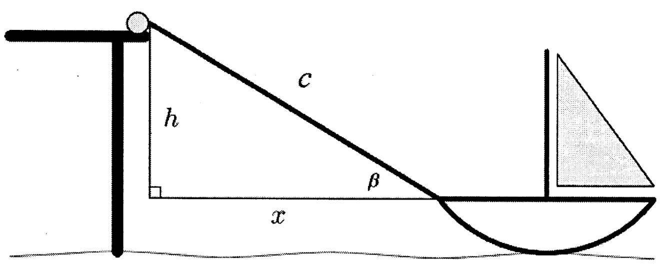
$\tan\theta = \frac{5}{9}$



$\sin\theta = \frac{5}{\sqrt{106}}$

8. (5 points) A boat is being pulled toward a dock as shown in the figure below. Determine the angle β if $x = 6\,\text{m}$ and $h = 2\,\text{m}$. Give your answer in degree measure, rounded to the nearest hundredth of a degree.

$\tan\beta = \frac{2}{6}$



$\beta = \tan^{-1}\left(\frac{2}{6}\right)$

$\approx 18.43°$

9. (6 points) Verify the identity: $\dfrac{\sec^2\theta - 1}{\sec^2\theta} = \sin^2\theta$

$$\frac{\sec^2\theta - 1}{\sec^2\theta} = \frac{\sec^2\theta}{\sec^2\theta} - \frac{1}{\sec^2\theta}$$

$$= 1 - \cos^2\theta = \sin^2\theta$$

✓

10. (8 points) Find the exact solutions in the interval $[0, 2\pi)$: $2\sin^2 x - 3\sin x + 1 = 0$

$u = \sin x$

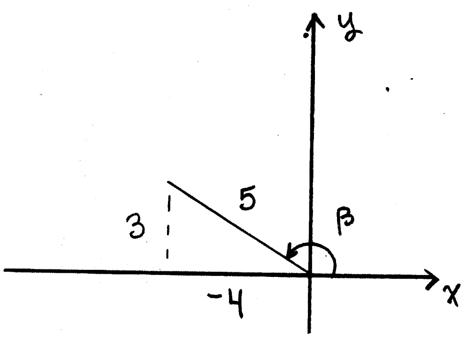
$2u^2 - 3u + 1 = 0 \Rightarrow (2u-1)(u-1) = 0$

$u = \frac{1}{2}$ or $u = 1$

$\sin x = \frac{1}{2}$ $\qquad$ $\sin x = 1$

$\boxed{x = \frac{\pi}{6} \text{ or } x = \frac{5\pi}{6} \text{ or } x = \frac{\pi}{2}}$

11. (10 points) Given that β is a 2nd quadrant angle with $\cos\beta = -4/5$, find the exact values of $\sin 2\beta$, $\cos 2\beta$, and $\tan 2\beta$. Do not use a calculator for this problem.



$\cos\beta = -\frac{4}{5}$

$\sin\beta = \frac{3}{5}$

Double Angle Formulas

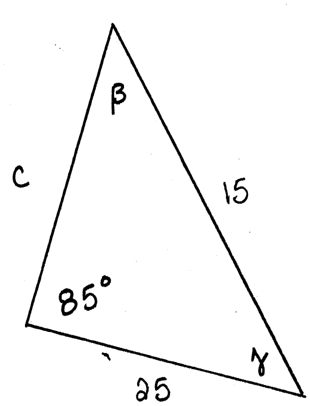
$\sin 2\beta = 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right) = -\frac{24}{25}$

$\cos 2\beta = \left(-\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{7}{25}$

$\tan 2\beta = \frac{-24/25}{7/25} = -\frac{24}{7}$

$\sqrt{5^2 - (-4)^2} = \sqrt{25-16}$
$= \sqrt{9} = 3$

12. (6 points) Solve the triangle: $\alpha = 85°$, $a = 15$, $b = 25$



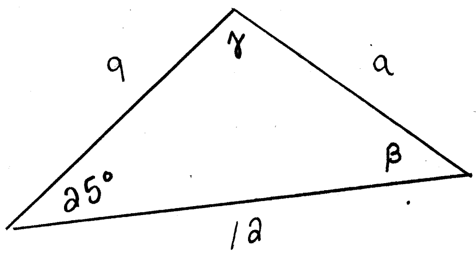
$\frac{\sin 85°}{15} = \frac{\sin\beta}{25}$

$\sin\beta = \underbrace{\frac{25 \sin 85°}{15} \approx 1.66}_{}$

No such angle β

$\boxed{\text{No solution.}}$

13. (10 points) Solve the triangle: $\alpha = 25°, \quad b = 9, \quad c = 12$



$a^2 = (9)^2 + (12)^2 - 2(9)(12)\cos 25°$

$\Rightarrow a^2 \approx 29.23752 \Rightarrow a \approx 5.41$

$(9)^2 = a^2 + (12)^2 - 2(a)(12)\cos\beta$

$\Rightarrow \cos\beta \approx 0.7107651 \Rightarrow \beta \approx 44.70°$

$\gamma = 180° - 25° - \beta \approx 110.30°$

14. (6 points) Let $z_1 = 3(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$ and $z_2 = 4(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$.

(a) Find the product $z_1 z_2$ in polar form.

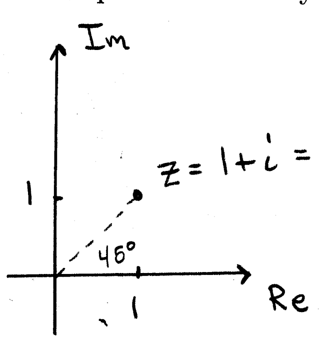
$z_1 \times z_2 = (3 \cdot 4)\left[\cos\left(\frac{\pi}{3} + \frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right]$

$= 12\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

(b) Write the product $z_1 z_2$ in standard form.

$12(0 + i) = 12i$

15. (6 points) Let $z = 1 + i$. Write z in polar form. Then use DeMoivre's theorem to compute z^6. Write your final answer in standard form.



$z = 1 + i = \sqrt{2}(\cos 45° + i\sin 45°)$

$\sqrt{1+1} = \sqrt{2}$

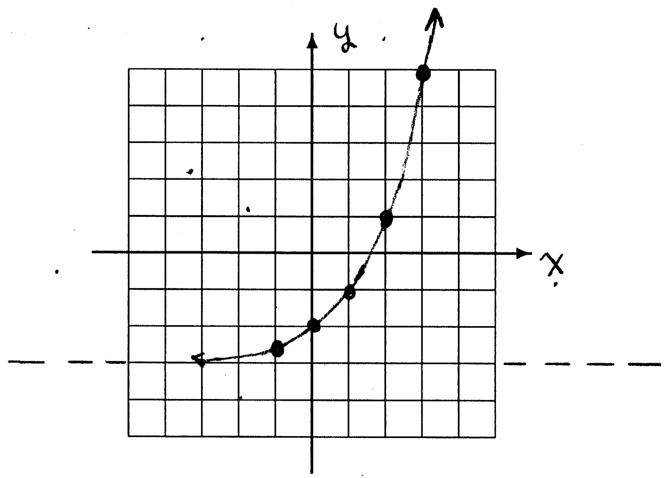
$z^6 = (\sqrt{2})^6\left(\cos(6 \cdot 45°) + i\sin(6 \cdot 45°)\right)$

$= 8(\cos 270° + i\sin 270°)$

$= 8(0 - i) = -8i$

16. (9 points) Determine five points on the graph of $f(x) = 2^x - 3$. Then plot your points and carefully sketch the graph. Indicate any asymptotes.

x	$y = 2^x - 3$
0	-2
1	-1
2	1
3	5
-1	2.5



H.A. $y = -3$

17. (2 points) Rewrite as a logarithmic equation: $4^5 = 1024$

$\log_4 1024 = 5$

18. (2 points) Rewrite as an exponential equation: $\log_7 343 = 3$

$7^3 = 343$

19. (4 points) Use properties of logarithms to completely expand: $\ln\left(\frac{x^4 y}{\sqrt{z^5}}\right)$

$4 \ln x + \ln y - \frac{5}{2} \ln z$

20. (3 points) Use the change-of-base formula to write $\log_7 50$ in terms of natural logarithms. Then use your calculator to compute the value. Round to the nearest thousandth.

$\log_7 50 = \frac{\ln 50}{\ln 7} = 2.010$

21. (8 points) Solve for x: $\log 5x + \log(x-1) = 2$

$$\log 5x(x-1) = 2$$

$$5x(x-1) = 100$$

$$x(x-1) = 20$$

$$x^2 - x - 20 = 0$$

$$(x-5)(x+4) = 0$$

$$\boxed{x = 5,} \quad \cancel{x = -4}$$

22. (8 points) Solve for x: $4^x = 5^{x^2}$

$$x \ln 4 = x^2 \ln 5$$

$$\boxed{x = 0 \quad \text{OR} \quad x = \frac{\ln 4}{\ln 5}}$$

23. (10 points) Polonium-210 has a half-life of 140 days. Use a model of the form $P(t) = Ae^{-kt}$ to determine the initial amount of Polonium-210 if there were 2.5 grams remaining after 600 days.

$$P(t) = P_0 e^{kt}$$

HALF-LIFE ... $\frac{1}{2} = e^{k(140)} \Rightarrow k = \frac{\ln \frac{1}{2}}{140}$

AFTER 600 DAYS ... $2.5 = P_0 e^{k(600)}$

$$P_0 = \frac{2.5}{e^{600k}} = \frac{2.5}{e^{600(\ln 1/2 / 140)}}$$

$$\approx \boxed{48.76 \text{ GRAMS}}$$

#4

$y = 3\cos(2x + \pi) = 3\cos 2(x + \frac{\pi}{2})$

Period $= \frac{2\pi}{2} = \pi$

Amp $= 3$

Shift $= \frac{\pi}{2}$ (left)