Math 130 - Quiz 6

October 21, 2020

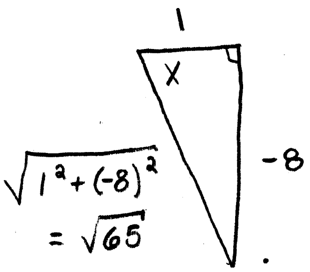
Name Key

Score

Show all work to receive full credit. Supply explanations when necessary. You must work individually on this quiz. This quiz is due October 26.

1. (2 points) Determine the exact values of $\sin(2x)$ and $\cos(2x)$ if x is the 4th quadrant angle with $\tan x = -8$.

$$\text{TAN } X = \frac{-8}{1} = \frac{\text{OPP}}{\text{ADJ}}$$



$$\text{SIN } X = \frac{-8}{\sqrt{65}}$$

$$\text{COS } X = \frac{1}{\sqrt{65}}$$

$$\text{SIN } 2X = 2 \text{ SIN } X \text{ COS } X$$

$$= 2\left(\frac{-8}{\sqrt{65}}\right)\left(\frac{1}{\sqrt{65}}\right) = \frac{-16}{65}$$

$$\text{COS } 2X = \text{COS}^2 X - \text{SIN}^2 X$$

$$= \frac{1}{65} - \frac{64}{65} = \frac{-63}{65}$$

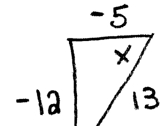
2. (2 points) Use a half-angle formula to determine the exact value of $\cos(7\pi/8)$.

$$\underbrace{\frac{7\pi}{8} = \frac{1}{2}\left(\frac{7\pi}{4}\right), \quad \text{COS } \frac{X}{2} = \pm\sqrt{\frac{1+\text{COS } X}{2}}, \quad \frac{7\pi}{8} \text{ IS IN QUAD 2}}$$

$$\text{COS}\left(\frac{1}{2}\frac{7\pi}{4}\right) = -\sqrt{\frac{1+\text{COS }\frac{7\pi}{4}}{2}}$$

$$= -\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = -\sqrt{\frac{2+\sqrt{2}}{4}}$$

$$\approx -0.9239$$



$5^2 + 12^2 = 169 = 13^2$

$\cos x = -\frac{5}{13}$

3. (2 points) Suppose that x is the 3rd quadrant angle with $\sin x = -\frac{12}{13}$. Use a half-angle formula to find the exact value of $\sin(x/2)$.

$180° < x < 270° \Rightarrow 90° < \frac{x}{2} < 135° \Rightarrow \frac{x}{2}$ IS IN 2ND QUAD.

$\sin \frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}} = +\sqrt{\frac{1-\left(-\frac{5}{13}\right)}{2}} = \sqrt{\frac{18}{26}} = \boxed{\sqrt{\frac{9}{13}}}$

USE + BECAUSE SINE IS POS IN 2ND QUAD

4. (1 point) Use a power-reducing formula to rewrite $\sin^2(6x)$.

$\sin^2 x = \frac{1-\cos 2x}{2} \Rightarrow \boxed{\sin^2(6x) = \frac{1-\cos(12x)}{2}}$

5. (1 point) Use a double-angle formula to rewrite $4\sin(8x)\cos(8x)$ in terms of a single trig function.

$\sin 2x = 2\sin x \cos x$

REPLACE X BY 8X AND MULT. BY 2 $\Rightarrow$ $\boxed{2\sin 16x = 4\sin 8x \cos 8x}$

6. (2 points) Use a product-to-sum formula to rewrite $10\cos(5x)\sin(10x)$.

$10\cos 5x \sin 10x$

$= 10\left(\frac{1}{2}\right)\left[\sin(10x+5x) + \sin(10x-5x)\right]$

$= \boxed{5\left(\sin(15x) + \sin(5x)\right)}$