Math 130 - Review 1

[redacted]

Name key

Score ________

These problems may help you review for Test 1. Your actual test will not be as long as this review packet.

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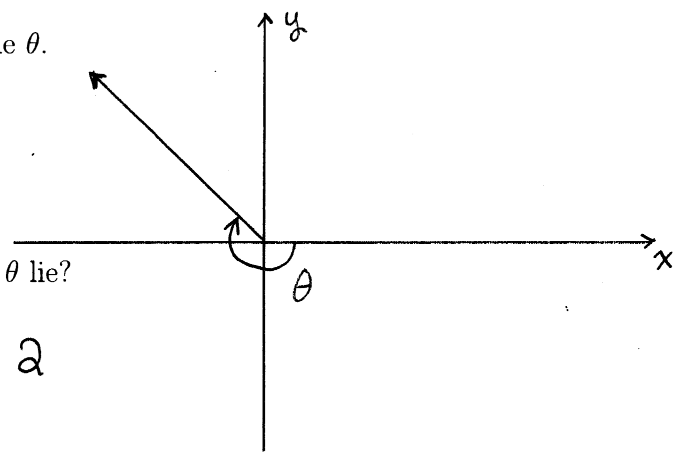
1. Use radian measure to say what it means for an angle to be *acute.*

   THE ANGLE θ IS ACUTE IF $0 < \theta < \frac{\pi}{2}$.

2. The angle θ lies in standard position and has radian measure $-5\pi/4$.

   (a) Roughly sketch the angle θ.



   (b) In which quadrant does θ lie?

   QUADRANT 2

   (c) Determine the degree measure of θ.

   $-\frac{5\pi}{4} = -5(45°) = \boxed{-225°}$

   (d) Determine two (additional) coterminal angles and write in radian measure.

   $\boxed{\frac{3\pi}{4}}$ AND $\frac{3\pi}{4} + 2\pi = \boxed{\frac{11\pi}{4}}$

3. Determine the complement and the supplement of $\pi/12$. Write your answers in both radian and degree measure.

MAKE 90°

COMPLEMENT:

$\frac{\pi}{12}$, $\boxed{\frac{5\pi}{12}}$

$15°$, $\boxed{75°}$

MAKE 180°

SUPPLEMENT:

$\frac{\pi}{12}$, $\boxed{\frac{11\pi}{12}}$

$15°$, $\boxed{165°}$

4. Convert 255° to radian measure.

$$255^\circ \cdot \frac{\pi}{180^\circ} = \frac{255}{180}\pi = \frac{17\pi}{12}$$

5. The minute hand of a clock is 8 in long.

(a) What angle does the hand sweep out in 20 minutes?

20 MIN IS $\frac{1}{3}$ OF AN HOUR $\Rightarrow$ $\frac{1}{3}$ OF 360° = $120^\circ = \frac{2\pi}{3}$

(b) What arc length is swept out in 20 minutes?

$$(8)\left(\frac{2\pi}{3}\right) = \frac{16\pi}{3}\text{ in} \approx 16.76\text{ in}$$

(c) Determine the angular speed of the minute hand.

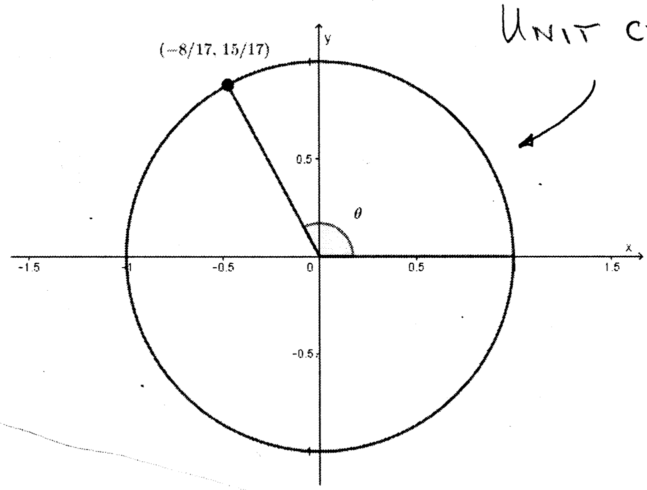
$\dfrac{120^\circ}{20\text{ min}} = 6^\circ$ per min OR $\dfrac{\frac{2\pi}{3}}{20\text{ min}} = \dfrac{\pi}{30}$ RADIANS/min

(d) Determine the linear speed of the tip of the minute hand.

$$v = r\omega = (8\text{ in})\left(\frac{\pi}{30\text{ min}}\right) = \frac{8\pi}{30}\text{ in/min} = \frac{4\pi}{15}\text{ in/min}$$

6. Find the exact values of the six trigonometric functions at θ. Write your answers as fractions in lowest terms.

UNIT CIRCLE!



$\cos\theta = \dfrac{-8}{17}$

$\sin\theta = \dfrac{15}{17}$

$\tan\theta = -\dfrac{15}{8}$

$\sec\theta = -\dfrac{17}{8}$

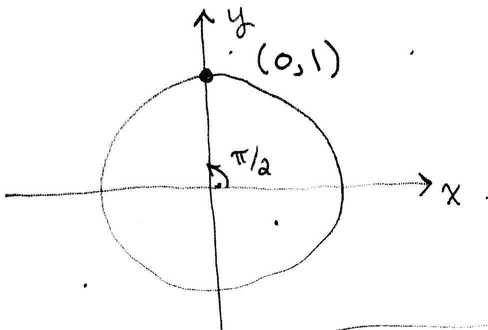
$\csc\theta = \dfrac{17}{15}$

$\cot\theta = -\dfrac{8}{15}$

7. Find the exact coordinates of the point (x, y) on the unit circle that corresponds to the angle θ. Do not use a calculator.

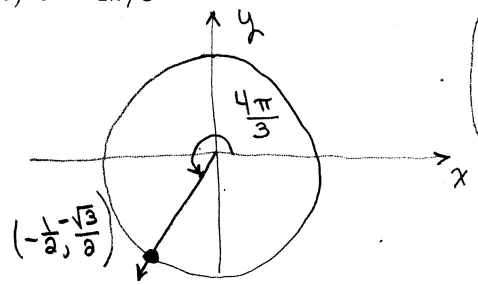
(a) $\theta = \pi/2$

$\boxed{(0,1)}$



(b) $\theta = 4\pi/3$

60° REFERENCE ANGLE



$\boxed{\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)}$

8. Briefly explain why $\sin(13\pi/6) = \sin(\pi/6)$.

SINE HAS PERIOD 2π...

$$\sin\left(\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6} + 2\pi\right) = \sin\left(\frac{13\pi}{6}\right)$$

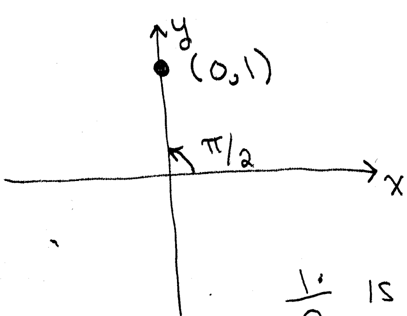
9. If $\sin t = \frac{1}{2}$, then what are the values of $\sin(-t)$ and $\csc(-t)$?

SINE IS AN ODD FUNCTION: $\sin(-x) = -\sin x$

$$\sin(-t) = -\sin t = \boxed{-\frac{1}{2}}, \quad \csc(-t) = \frac{1}{\sin(-t)} = \boxed{-2}$$

10. Without using your calculator, determine the value of $\tan(\pi/2)$. Suppose your classmate used a calculator to compute the value and got 0.0274224. What did your classmate do wrong?

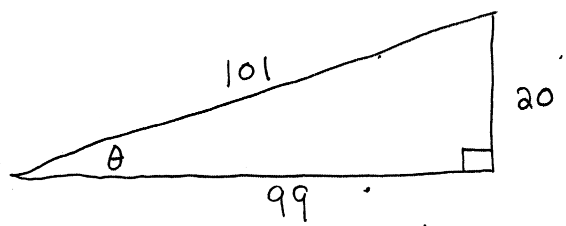
CALCULATOR IS IN DEGREE MODE!



$\frac{1}{0}$ IS NOT DEFINED.

TAN $\frac{\pi}{2}$ IS NOT DEFINED.

11. A right triangle has sides of lengths 20, 99, and 101. Let θ be smallest angle. Find the values of the six trigonometric functions at θ.



$\sin\theta = \frac{20}{101}$ $\quad \csc\theta = \frac{101}{20}$

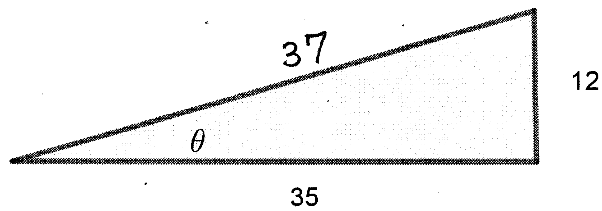
$\cos\theta = \frac{99}{101}$ $\quad \sec\theta = \frac{101}{99}$

$\tan\theta = \frac{20}{99}$ $\quad \cot\theta = \frac{99}{20}$

12. Refer to the right triangle shown below. Find the values of the six trigonometric functions at θ.

$12^2 + 35^2 = 1369$

$= 37^2$



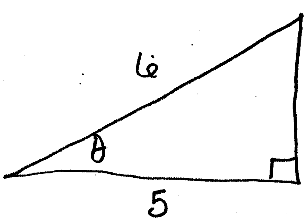
$\sin\theta = \frac{12}{37}$ $\quad \csc\theta = \frac{37}{12}$

$\cos\theta = \frac{35}{37}$ $\quad \sec\theta = \frac{37}{35}$

$\tan\theta = \frac{12}{35}$ $\quad \cot\theta = \frac{35}{12}$

$\sec\theta = \frac{HYP}{ADJ}$

13. Sketch a right triangle with an acute angle θ such that $\sec\theta = \frac{6}{5}$. Then find the values of the six trigonometric functions at θ.



$\sqrt{36-25} = \sqrt{11}$

$\sin\theta = \frac{\sqrt{11}}{6}$ $\quad \csc\theta = \frac{6}{\sqrt{11}}$

$\cos\theta = \frac{5}{6}$ $\quad \sec\theta = \frac{6}{5}$

$\tan\theta = \frac{\sqrt{11}}{5}$ $\quad \cot\theta = \frac{5}{\sqrt{11}}$

14. Is it true that $\frac{\sin 60°}{\sin 30°} = \sin 2°$? If someone thinks so, what mistake are they probably making?

$\underbrace{\frac{\sin 60°}{\sin 30°}}_{\text{No way!}}$

$$\frac{\sin 60°}{\sin 30°} \neq \sin\left(\frac{60°}{30°}\right)$$

15. Based only on your memory, write the values of each of the following.

(a) $\sin 30°$

$$= \boxed{\frac{1}{2}}$$

(b) $\cos(\pi/3)$

$$= \boxed{\frac{1}{2}}$$

(c) $\tan(\pi/4)$ $= \dfrac{\sqrt{2}/2}{\sqrt{2}/2} = \boxed{1}$

16. Use trig identities to transform one side of the equation into the other.

(a) $\cot\alpha \sin\alpha = \cos\alpha$

$$\left(\frac{\cos\alpha}{\cancel{\sin\alpha}}\right)\cancel{\sin\alpha} = \cos\alpha$$

(b) $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$$\sec^2\theta - \tan^2\theta$$

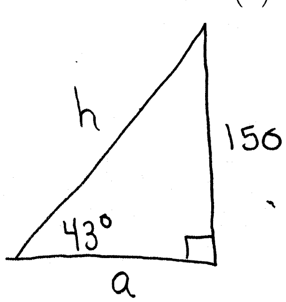
$$\left(\tan^2\theta + 1\right) - \tan^2\theta = 1$$

17. A guy wire runs from the ground to a cell tower. The wire is attached to the tower 150 feet above the ground, and the angle formed between the wire and the ground is 43°. Assume that the tower makes a right angle with the ground.

(a) How long is the guy wire?

$$\sin 43° = \frac{150}{h} \Rightarrow h = \frac{150\text{ FT}}{\sin 43°} \approx \boxed{219.9\text{ FT}}$$

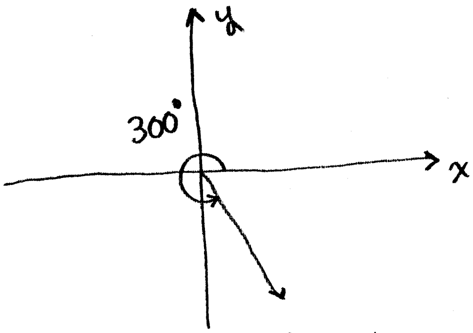
(b) How far is the base of the tower from the point on the ground where the guy wire is anchored?



$$\tan 43° = \frac{150}{a} \Rightarrow a = \frac{150\text{ FT}}{\tan 43°}$$

$$\approx \boxed{160.9\text{ FT}}$$

18. $\theta = 300°$. Determine the reference angle. Without using your calculator or unit circle, determine the values of the six trigonometric functions at θ.



$\boxed{\text{REF } \angle \text{ IS } 60°}$

$\sin 300° = -\sin 60° = -\frac{\sqrt{3}}{2}$

$\cos 300° = \cos 60° = \frac{1}{2}$

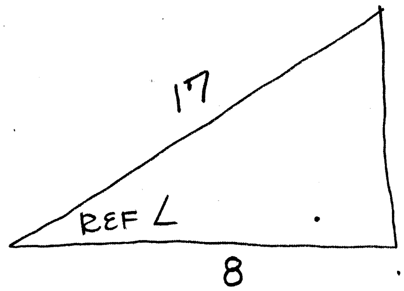
$\tan 300° = -\sqrt{3}$

$\csc 300° = -\frac{2}{\sqrt{3}}$

$\sec 300° = 2$

$\cot 300° = -\frac{1}{\sqrt{3}}$

19. $\tan\theta = 15/8$ and $\sin\theta < 0$.
Find the exact values of the six trigonometric functions at θ.



$15^2 + 8^2 = 225 + 64$
$= 289$

USING THE REF ∠
AND THAT
$\tan\theta > 0$, $\sin\theta < 0$

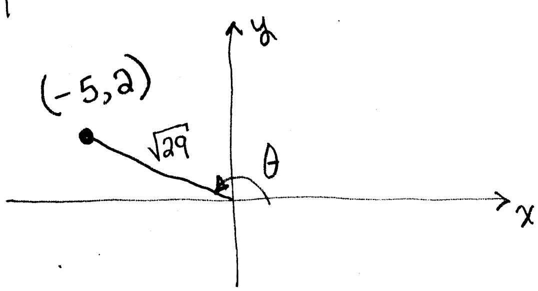
$\sin\theta = -\frac{15}{17}$ $\quad \csc\theta = \frac{-17}{15}$

$\cos\theta = -\frac{8}{17}$ $\quad \sec\theta = -\frac{17}{8}$

$\tan\theta = \frac{15}{8}$ $\quad \cot\theta = \frac{8}{15}$

20. The point $(-5, 2)$ is on the terminal side of an angle in standard position. Find the exact values of the six trigonometric functions at that angle. Simplify your answers as much as possible.

$5^2 + 2^2 = 29$



$\sin\theta = \frac{2}{\sqrt{29}}$ $\quad \csc\theta = \frac{\sqrt{29}}{2}$

$\cos\theta = -\frac{5}{\sqrt{29}}$ $\quad \sec\theta = -\frac{\sqrt{29}}{5}$

$\tan\theta = -\frac{2}{5}$ $\quad \cot\theta = -\frac{5}{2}$

21. Determine the quadrant in which θ lies.

(a) $\sin\theta < 0, \quad \cos\theta < 0$

QUAD 3

(b) $\sec\theta > 0, \quad \cot\theta < 0$

$\cos\theta > 0$, $\tan\theta < 0$

$\sin\theta < 0$

QUAD 4

22. Determine the period and amplitude.

(a) $y = 5\sin 2x$

Amplitude = $\boxed{5}$, Period = $\frac{2\pi}{2}$ = $\boxed{\pi}$

(b) $y = -8\cos 100\pi x$

Amplitude = $\boxed{8}$, Period = $\frac{2\pi}{100\pi}$ = $\boxed{\frac{1}{50}}$

23. Describe how the graph of each equation below can be obtained from the graph of $y = \sin x$.

(a) $y = -3\sin x$

Flip the graph about the x-axis and vertically stretch to have amplitude 3.

(b) $y = \sin(x - \pi)$

Shift right π units.

(c) $y = 2 + \sin \pi x$

Rescale/Compress to period 2 and shift up 2 units.

24. On the attached graph paper, sketch the graph of $y = -1 + \cos 4\pi x$. (Include two full periods.)

See attached.

25. On the attached graph paper, sketch the graph of $y = -\sin\left(\pi x + \frac{\pi}{4}\right)$. (Include two full periods.)

SEE ATTACHED.

26. Write an equation whose graph has the given characteristics.

(a) A sine curve with period π, an amplitude of 2, a right phase shift of $\pi/2$, and a vertical translation up 1 unit.

$$y = 1 + 2 \sin 2\left(x - \frac{\pi}{2}\right)$$

OR

$$y = 1 + 2 \sin (2x - \pi)$$

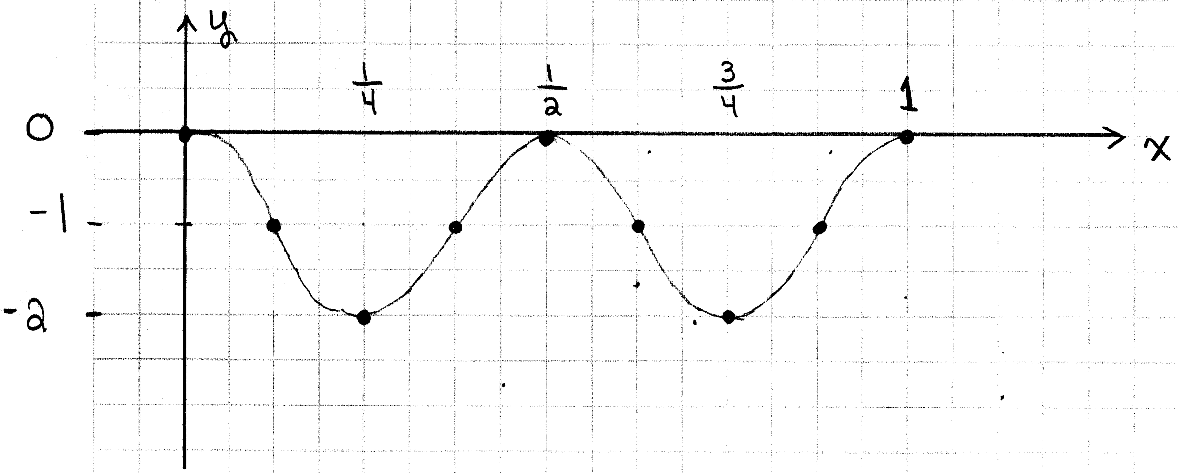
(b) A cosine curve with period 4π, an amplitude of 3, a left phase shift of $\pi/2$, and a vertical translation down 2 units.

$$y = -2 + 3 \cos \frac{1}{2}\left(x + \frac{\pi}{2}\right)$$

OR

$$y = -2 + 3 \cos \left(\frac{1}{2}x + \frac{\pi}{4}\right)$$

#24 $y = -1 + \cos 4\pi x$ Period is $\frac{2\pi}{4\pi} = \frac{1}{2}$



$y = -\sin\left(\pi x + \frac{\pi}{4}\right) = -\sin \pi\left(x + \frac{1}{4}\right)$, Period $= \frac{2\pi}{\pi} = 2$

#25