Math 130 - Test 1
September 16, 2020

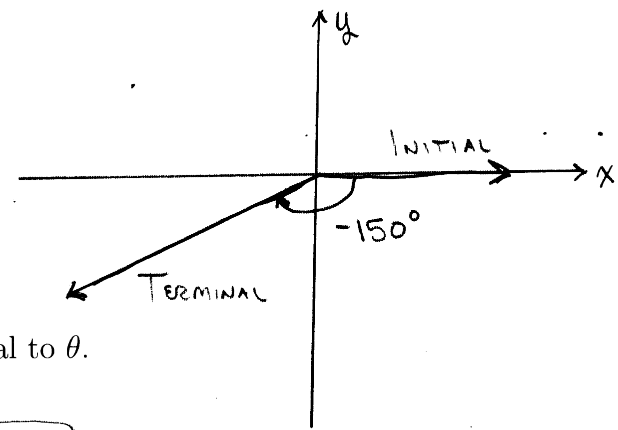
Name Key

Score ________

Show all work to receive full credit. Supply explanations where necessary. With the exception of rationalizing denominators, simplify all answers.

1. (6 points) Consider the angle θ whose degree measure is $-150°$.

   (a) Sketch the angle θ in standard position. Label the initial and terminal sides.



   (b) Determine a different angle that is coterminal to θ.

$$360° - 150° = \boxed{210°}$$

   (c) Convert $-150°$ to radian measure.

$$-150° \times \frac{\pi}{180°} = \boxed{-\frac{5\pi}{6}}$$

2. (5 points) Find the length of the arc of a circle of radius 10 centimeters subtended by the central angle of 50°. Round your answer to the nearest hundredth.

$S = r\theta$ WHEN θ IS IN RADIANS.

$$\theta = 50° \times \frac{\pi}{180°} = \frac{5\pi}{18}$$

ARC LENGTH $= (10)\left(\frac{5\pi}{18}\right)$

$\approx \boxed{8.73 \text{ cm}}$

3. (6 points) A wheel of radius 8 inches is rotating at 3 radians per second.

(a) Find the linear speed in inches per second.

$V = r\omega$, WHERE ω = ANG. SPEED IN RADIANS/TIME

$V = (8 \text{ in})(3 \text{ RAD}/s) = \boxed{24 \text{ in}/s}$

(b) Find the angular speed in degrees per second.

$\omega = (3 \text{ RAD}/s)\left(\frac{180°}{\pi \text{ RAD}}\right) = \frac{540°}{\pi}$ PER SECOND $\approx 171.89°$ PER SEC

(c) Find the angular speed in revolutions per minute (RPM).

$\omega = \left(3 \text{ RAD}/s\right)\left(\frac{1 \text{ REV}}{2\pi \text{ RAD}}\right)\left(\frac{60 s}{1 \text{ min}}\right) = \frac{180}{2\pi}$ REV/min

≈ 28.65 RPM

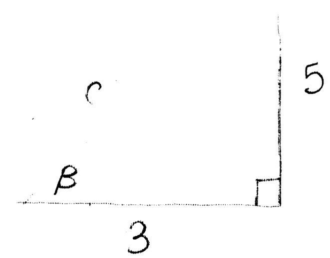
4. (8 points) The two legs of a right triangle have lengths 3 and 5.

(a) Determine the length of the hypotenuse.

$c^2 = 9 + 25$

$= 34$

$\boxed{c = \sqrt{34}}$



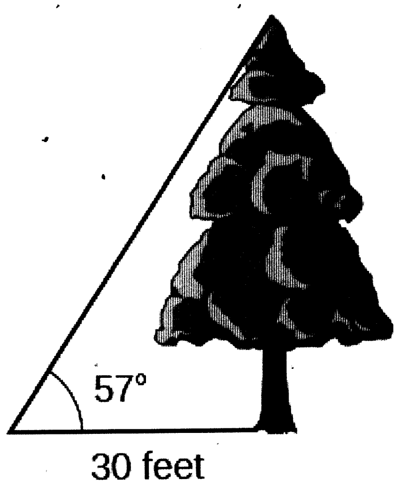
(b) Let β be the angle opposite the leg of length 5. Determine the exact values of the six trigonometric functions at β. You do not have to rationalize your denominators, but otherwise write your fractions as simple as possible.

$\sin \beta = \frac{5}{\sqrt{34}}$ $\quad \csc \beta = \frac{\sqrt{34}}{5}$

$\cos \beta = \frac{3}{\sqrt{34}}$ $\quad \sec \beta = \frac{\sqrt{34}}{3}$

$\tan \beta = \frac{5}{3}$ $\quad \cot \beta = \frac{3}{5}$

5. (5 points) To find the height of a tree, a person walks to a point 30 feet from the base of the tree. She measures an angle of 57° between a line of sight to the top of the tree and the ground (see figure). Find the height of the tree. Round to the nearest tenth of a foot.



$\tan 57^\circ = \dfrac{\text{Height}}{30}$

$\text{Height} = 30 \tan 57^\circ$

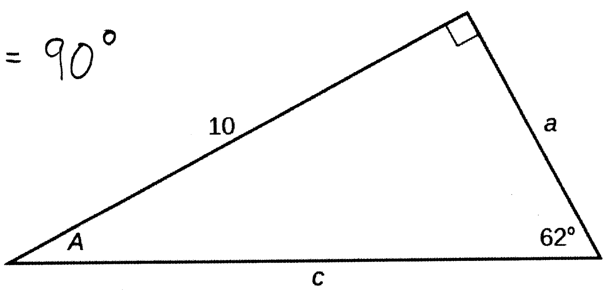
$\approx \boxed{46.2 \text{ FT}}$

6. (8 points) Complete the triangle by finding the angle A and the side lengths a and c. Round the lengths to 4 decimal places.

$m(\angle A) + 62^\circ = 90^\circ$

$\Downarrow$

$\boxed{m(\angle A) = 28^\circ}$



$\tan 28^\circ = \dfrac{a}{10}$

$\Rightarrow \boxed{a = 10 \tan 28^\circ \approx 5.3171}$

$\sin 62^\circ = \dfrac{10}{c}$

$\Downarrow$

$\boxed{c = \dfrac{10}{\sin 62^\circ} \approx 11.3257}$

7. (6 points) For each part below, use the information to determine the quadrant in which θ lies.

(a) $\sin\theta < 0$, $\cos\theta > 0$

3 or 4   1 or 4 $\rightarrow$ $\boxed{4^{\text{TH}} \text{ QUAD}}$

(b) $\csc\theta < 0$, $\cot\theta > 0$

3 or 4   1 or 3 $\rightarrow$ $\boxed{3^{\text{RD}} \text{ QUAD}}$

(c) $\tan\theta < 0$, $\sec\theta < 0$

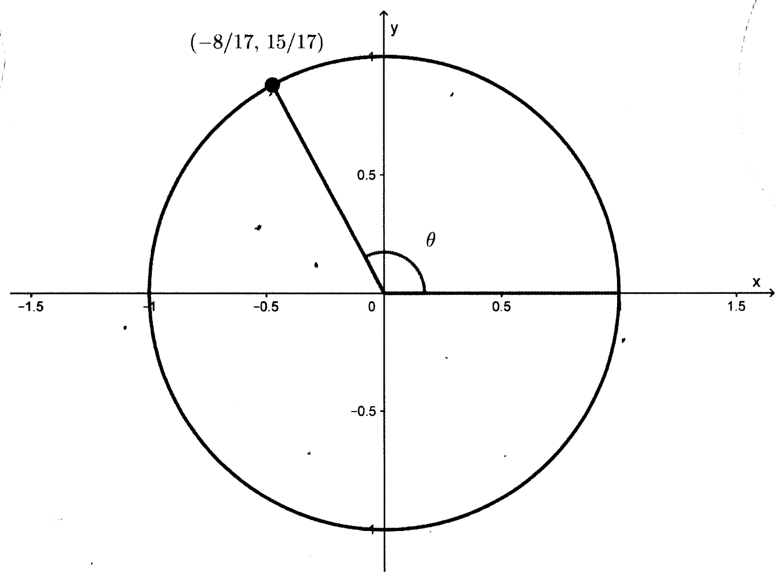
2 or 4   2 or 3 $\rightarrow$ $\boxed{2^{\text{ND}} \text{ QUAD}}$

8. (6 points) Find the exact values of the six trigonometric functions at θ. Write your answers as fractions in lowest terms.

$\sin\theta = \frac{15}{17}$

$\cos\theta = -\frac{8}{17}$

$\tan\theta = -\frac{15}{8}$



$\csc\theta = \frac{17}{15}$

$\sec\theta = -\frac{17}{8}$

$\cot\theta = -\frac{8}{15}$

9. (6 points) For each angle, compute the reference angle.

(a) $-150°$ 30° $180° - 150° = 30°$

(b) $7\pi/4$ $2\pi - \frac{7\pi}{4} = \frac{\pi}{4}$

(c) $3\pi/4$ 45° $\pi - \frac{3\pi}{4} = \frac{\pi}{4}$

10. (5 points) Describe how you would use the reference angle and the table shown below to determine $\sin 300°$.

θ	0°	30°	45°	60°	90°
$\sin\theta$	0	1/2	$\sqrt{2}/2$	$\sqrt{3}/2$	1

300° 60°

300° IS IN THE 4TH QUAD.

THE REFERENCE ANGLE IS 60°. $\sin 60° = \frac{\sqrt{3}}{2}$

IN 4TH QUAD, THE SINE IS NEG.

$\sin(300°) = -\sin(60°) = -\frac{\sqrt{3}}{2}$

11. (4 points) Pretend that $\sin 53° = 0.8512$ (it's not, but pretend!). Based on that (fake) value of $\sin 53°$, what would be the (fake) value of $\cos 37°$? Briefly explain your reasoning.

(FAKE) $\cos 37° = 0.8512$ BECAUSE COFUNCTIONS AT COMPLEMENTARY ANGLES ARE EQUAL.

12. (5 points) Use basic trigonometric identities to simplify each expression.

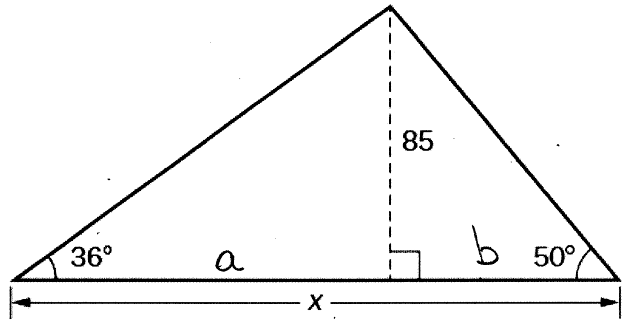
(a) $\csc t \tan t \cos t$

$$\frac{1}{\cancel{\sin t}} \times \frac{\cancel{\sin t}}{\cancel{\cos t}} \times \frac{\cancel{\cos t}}{1} = \boxed{1}$$

(b) $1 - \cos^2\theta = \boxed{\sin^2\theta}$

BECAUSE $\sin^2\theta + \cos^2\theta = 1$

13. (6 points) Compute x. Round to two decimal places.



$\tan 36° = \dfrac{85}{a}$

$a = \dfrac{85}{\tan 36°}$

$\tan 50° = \dfrac{85}{b}$

$b = \dfrac{85}{\tan 50°}$

$x = a + b$

$= \dfrac{85}{\tan 36°} + \dfrac{85}{\tan 50°}$

$\approx \boxed{188.32}$

14. (8 points) On the attached graph paper, sketch the graph of $y = \sin x$. Include two full periods. Label your axes.

Use graph paper. SEE ATTACHED GRAPH.

15. (8 points) Suppose you were given the graph of $y = \cos x$. Describe exactly how the graph of each of the following would be different.

(a) $y = 2 + \cos x$

THE GRAPH OF $y = \cos X$ IS SHIFTED UP 2 UNITS

(b) $y = 2\cos x$

THE AMPLITUDE IS 2 INSTEAD OF 1. THE GRAPH OF $y = \cos X$ IS STRETCHED VERTICALLY BY A FACTOR OF 2.

(c) $y = \cos(2x)$

THE PERIOD IS $\frac{2\pi}{2} = \pi$ RATHER THAN 2π. THE GRAPH OF $y = \cos X$ IS COMPRESSED HORIZONTALLY.

(d) $y = -2\cos x$

GRAPH IS THE SAME AS IN (b) EXCEPT FLIPPED ABOUT THE X-AXIS.

16. (3 points) What is the period of the graph of the equation $y = 1 + 2\sin(3x + 4)$?

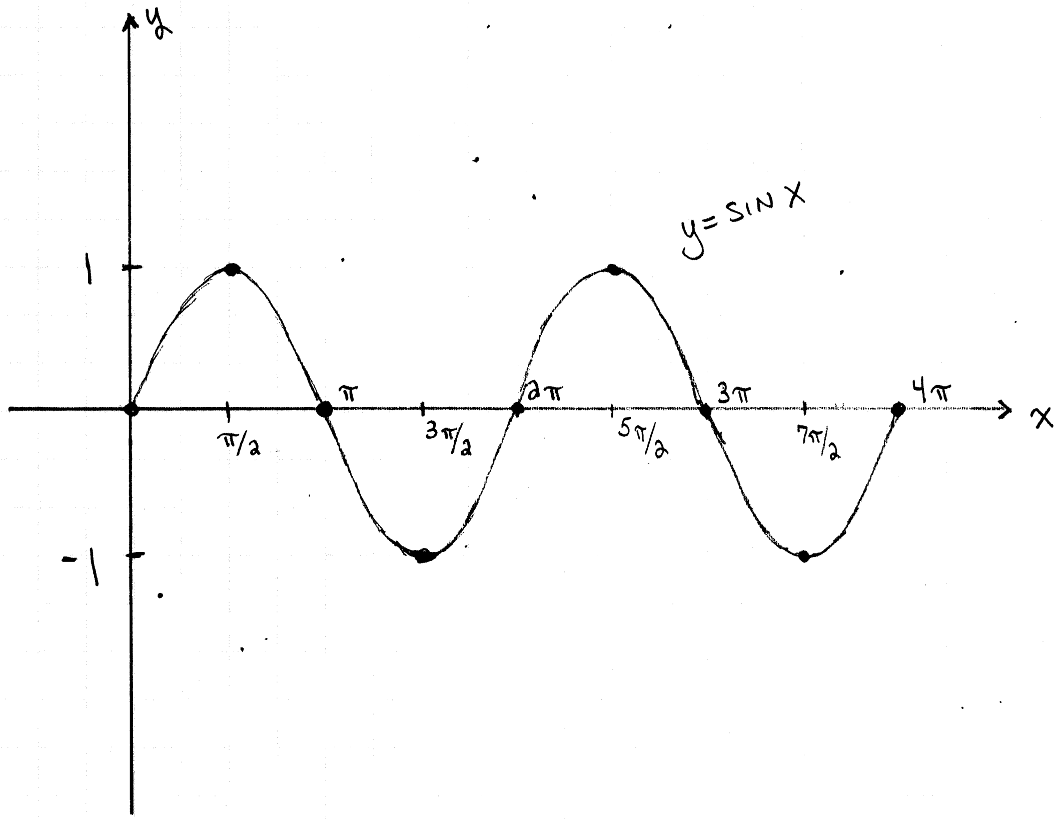
$$\boxed{\frac{2\pi}{3}}$$

17. (3 points) What would be the y-coordinate of the highest points on the graph of $y = 1 + 2\sin x$? Briefly explain how you know.

FOR $y = \sin X$, THE HIGHEST PTS ARE AT $y = 1$.

FOR $y = 2\sin X$, " " " WOULD BE AT $y = 2$.

FOR $y = 1 + 2\sin X$, " " " " " " $y = 3$.



x	$\sin x$
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0
$\frac{5\pi}{2}$	1
3π	0
$7\pi/2$	-1
4π	0