Math 130 - Test 2

October 14, 2020

Name key

Score

Show all work to receive full credit. Supply explanations where necessary. When finding exact answers, simplify as much as possible. You may use your unit circle and trig identity card on any problem unless otherwise indicated.

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1. (10 points [5]) Think about how the graph of $y = 2 + 3\sin(x - \pi)$ is related to the graph of $y = \sin x$.

   (a) List the sequence of transformations (in order) that transform the graph of $y = \sin x$ to that of $y = 2 + 3\sin(x - \pi)$.

   START WITH THE GRAPH OF $y = \sin x$. THEN

   ① SHIFT RIGHT π UNITS (PHASE SHIFT)

   ② VERTICALLY STRETCH BY A FACTOR OF 3 (AMPLITUDE CHANGE)

   ③ VERTICALLY SHIFT 2 UNITS UP.

   END WITH THE GRAPH OF $y = 2 + 3\sin(x - \pi)$.

   (b) The point $(\pi/2, 1)$ is on the graph of $y = \sin x$. How is that point moved by the transformations? Describe and give the coordinates of the new point.

   START AT $\left(\frac{\pi}{2}, 1\right)$ ① SHIFT RIGHT π UNITS TO $\left(\pi + \frac{\pi}{2}, 1\right)$ OR $\left(\frac{3\pi}{2}, 1\right)$.

   ② VERTICALLY STRETCH BY FACTOR OF 3 $\Rightarrow \left(\frac{3\pi}{2}, 3\right)$

   ③ SHIFT UP TWO UNITS $\Rightarrow \boxed{\left(\frac{3\pi}{2}, 5\right)}$

   (c) What is an equation of the midline for the graph of $y = 2 + 3\sin(x - \pi)$?

   $\boxed{y = 2}$

2. (10 points [5]) Consider the graph of $y = -2\cos(\pi x)$.

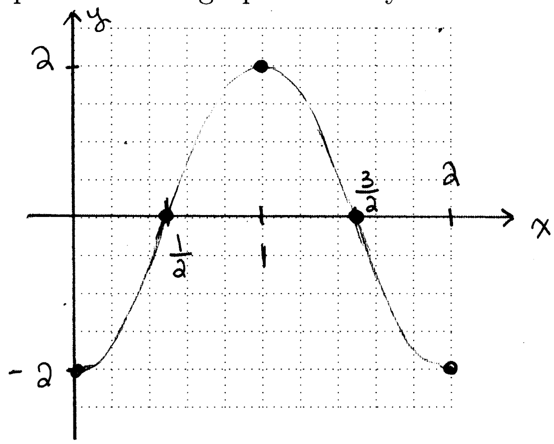
(a) What are the (i) amplitude, (ii) period, and (iii) midline for the graph.

(i) AMPLITUDE $= |-2| = \boxed{2}$

(ii) PERIOD $= \frac{2\pi}{\pi} = 2$

(iii) MIDLINE: $y = 0$

(b) Sketch exactly one full period of the graph. Label your axes.



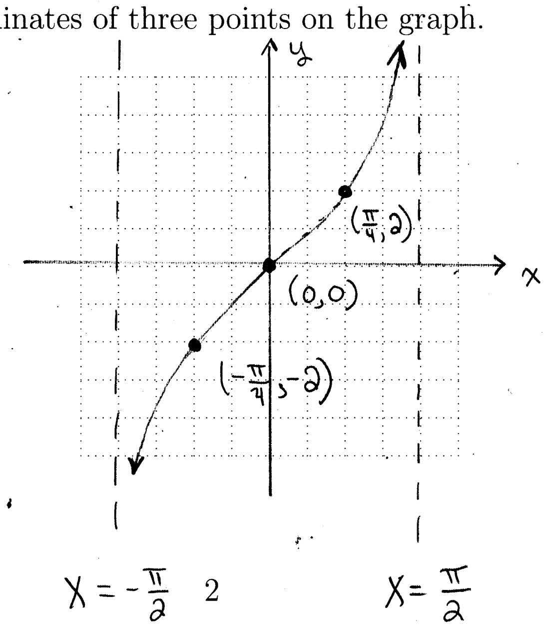
(c) Give the exact coordinates of the (i) highest points, (ii) lowest points, and (iii) x-intercepts of your graph.

(i) $(1, 2)$

(ii) $(0, -2)$, $(2, -2)$

(iii) $(\frac{1}{2}, 0)$, $(\frac{3}{2}, 0)$

3. (8 points [5]) Sketch one period of the graph of $y = 2\tan x$. Label your asymptotes, and give the exact coordinates of three points on the graph.



4. (8 points [5]) Determine the locations of two consecutive asymptotes of the graph of $y = 1 + 5\cot(2x + \frac{\pi}{4})$.

$y = \cot x$ HAS TWO CONSEC. V. A.'S

AT $x = 0$ AND $x = \pi$.

So...

$2x + \frac{\pi}{4} = 0$

$x = -\frac{\pi}{8}$

$2x + \frac{\pi}{4} = \pi$

$2x = \frac{3\pi}{4}$

$x = \frac{3\pi}{8}$

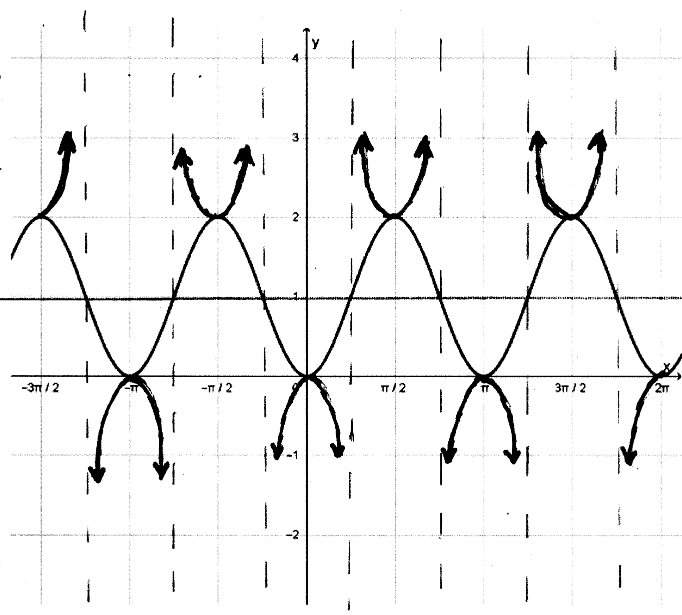
5. (5 points [5]) Shown below is the graph of $y = 1 - \sin(2x + \frac{\pi}{2})$. Use the given graph to sketch the graph of $y = 1 - \csc(2x + \frac{\pi}{2})$. Clearly indicate any special features of the graph such as asymptotes, intercepts, etc.

ASYMPTOTES OCCUR WHERE

$\sin(2x + \frac{\pi}{2}) = 0$

OR WHERE

$1 - \sin(2x + \frac{\pi}{2}) = 1$



MIDLINE $y = 1$

6. (2 points [1,10]) Determine the exact value of $\sin^{-1}(\sin(15\pi/4))$.

SUBTRACT $4\pi = \frac{16\pi}{4}$

$\sin^{-1}\left(\sin\left(\frac{15\pi}{4}\right)\right) = \sin^{-1}\left(\sin\left(-\frac{\pi}{4}\right)\right)$

$= -\frac{\pi}{4}$

7. (10 points [1,10]) Use your knowledge of the values of the trigonometric functions at special angles to determine the exact value of each of the following. Do not use a calculator.

(a) $\arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ BECAUSE $\frac{\pi}{3}$ IS IN $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ AND $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

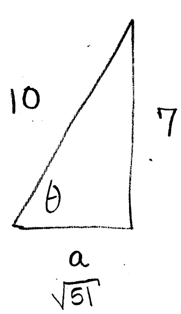
(b) $\tan^{-1}(-1) = -\frac{\pi}{4}$ BECAUSE $-\frac{\pi}{4}$ IS IN $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ AND $\tan\left(-\frac{\pi}{4}\right) = -1$

(c) $\arccos\left(\frac{-\sqrt{2}}{2}\right) = \frac{3\pi}{4}$ BECAUSE $\frac{3\pi}{4}$ IS IN $[0, \pi]$ AND $\cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

(d) $\cos^{-1}(2)$ DOES NOT EXIST BECAUSE COSINES ARE BETWEEN -1 AND 1

(e) $\sin^{-1}(1) = \frac{\pi}{2}$ BECAUSE $\frac{\pi}{2}$ IS IN $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ AND $\sin\left(\frac{\pi}{2}\right) = 1$

8. (6 points [1,2,10]) Use a right triangle to find the exact value of $\cot(\underbrace{\sin^{-1}(\tfrac{7}{10})}_{\theta})$.



$\sin\theta = \frac{7}{10}$

$a^2 + 7^2 = 10^2$

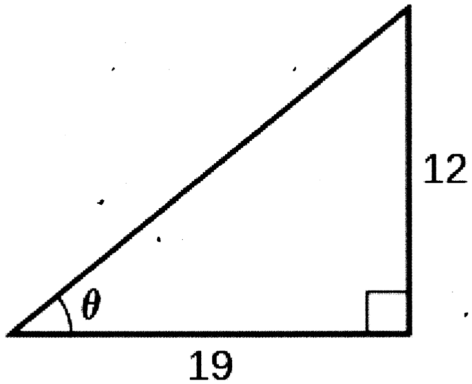
$\Downarrow$

$a^2 = 51$

$a = \sqrt{51}$

$\cot\theta = \frac{\sqrt{51}}{7}$

9. (3 points [1,2,10]) Determine the angle θ. Give your answer in degree measure, rounded to the nearest tenth of a degree.



$$\tan \theta = \frac{12}{19}$$

$$\Downarrow$$

$$\theta = \tan^{-1}\left(\frac{12}{19}\right) \approx \boxed{32.3^\circ}$$

10. (3 points [3]) Simplify the expression: $\cos(\frac{\pi}{2} - x)\ \cos(-x)\ \csc x$

$$\cancel{(\sin x)}(\cos x)\left(\frac{1}{\cancel{\sin x}}\right) = \boxed{\cos x}$$

11. (3 points [3]) Verify the identity: $\dfrac{\sec^2 x - 1}{\sec^2 x} = \sin^2 x$

$$\frac{\sec^2 x}{\sec^2 x} - \frac{1}{\sec^2 x} = 1 - \cos^2 x = \sin^2 x \ \checkmark$$

12. (3 points [3]) Verify the identity: $(1 - \cos^2 \alpha)(1 + \cot^2 \alpha) = 1$

$$(\sin^2 \alpha)(\csc^2 \alpha)$$

$$= \cancel{\sin^2 \alpha}\left(\frac{1}{\cancel{\sin^2 \alpha}}\right) = 1 \ \checkmark$$

13. (3 points [3]) Simplify the expression: $\cos x \sin^2 x - \cos x$

$$\cos x\,(\sin^2 x - 1)$$

$$= (\cos x)(-1)(1 - \sin^2 x)$$

$$= (\cos x)(-1)(\cos^2 x) = \boxed{-\cos^3 x}$$

$\tan^2 x + 1$

14. (3 points [3]) Write with only tangents and factor: $\sec^2 x + 3\tan x + 1$

$$\tan^2 x + 1 + 3\tan x + 1$$

$$= \tan^2 x + 3\tan x + 2 = \boxed{(\tan x + 2)(\tan x + 1)}$$

15. (6 points [3]) Verify the identity: $\dfrac{\cos\theta}{1+\sin\theta} = \dfrac{1+\sin(-\theta)}{\cos(-\theta)}$

$$= \frac{1-\sin\theta}{\cos\theta} \cdot \frac{1+\sin\theta}{1+\sin\theta}$$

$$= \frac{1-\sin^2\theta}{\cos\theta\,(1+\sin\theta)} = \frac{\cos^2\theta}{\cos\theta\,(1+\sin\theta)}$$

$$= \frac{\cos\theta}{1+\sin\theta} \quad \checkmark$$

16. (7 points [3,6]) Write 105° as the sum or difference of two of our familiar angles. Then use the appropriate sum or difference formula to find the exact value of cos 105°. Do not use a calculator for this problem.

$$105° = 60° + 45°$$

$$\cos(60° + 45°) = \cos 60° \cos 45° - \sin 60° \sin 45°$$

$$= \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$$

17. (10 points [3,6]) Suppose a and b are first quadrant angles with $\sin a = \frac{4}{5}$ and $\cos b = \frac{1}{3}$. Compute $\sin(a+b)$. (Hint: You will probably need to use a Pythagorean identity.)

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$= \left(\frac{4}{5}\right)\left(\frac{1}{3}\right) + (?)(?) = \left(\frac{4}{5}\right)\left(\frac{1}{3}\right) + \left(\frac{3}{5}\right)\left(\frac{\sqrt{8}}{3}\right)$$

$$= \boxed{\frac{4+3\sqrt{8}}{15}}$$

a is in 1st quad with $\sin a = \frac{4}{5}$

$$\sin^2 a + \cos^2 a = 1$$

$$\Downarrow$$

$$\cos a = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{9}{25}} = 3/5$$

b is in 1st quad with $\cos b = \frac{1}{3}$

$$\sin^2 b + \cos^2 b = 1$$

$$\Downarrow$$

$$\sin b = \sqrt{1 - \frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{\sqrt{8}}{3}$$