Math 130 - Test 3A

November 18, 2020

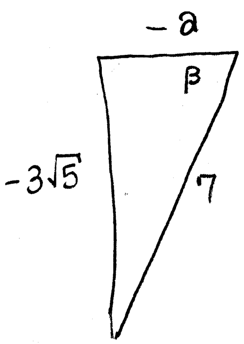
Name key

Score ________

Show all work to receive full credit. Supply explanations where necessary. You may use your unit circle and trig identity card on any problem. For each triangle described below, a is opposite α, b is opposite β, and c is opposite γ.

---

1. (7 points [3,6]) Given that β is a 3rd quadrant angle with $\cos\beta = -2/7$, find the exact values of $\sin 2\beta$ and $\cos 2\beta$. Do not use a calculator for this problem.



$$\cos\beta = -\frac{2}{7}$$

$$\sin\beta = -\frac{3\sqrt{5}}{7}$$

$$(-2)^2 + \text{opp}^2 = 7^2$$

$$\text{opp}^2 = 49 - 4$$

$$= 45$$

$$\text{opp} = -\sqrt{45} = -3\sqrt{5}$$

$$\sin 2\beta = 2\sin\beta\cos\beta$$

$$= 2\left(-\frac{3\sqrt{5}}{7}\right)\left(-\frac{2}{7}\right) = \boxed{\frac{12\sqrt{5}}{49}}$$

$$\cos 2\beta = \cos^2\beta - \sin^2\beta$$

$$= \left(-\frac{2}{7}\right)^2 - \left(-\frac{3\sqrt{5}}{7}\right)^2 = \frac{4}{49} - \frac{45}{49}$$

$$= \boxed{-\frac{41}{49}}$$

2. (8 points [9]) Find the exact solutions: $\cos^2 x - \cos x = 0$
(Find all solutions.)

$\cos^2 x - \cos x = 0$

$\cos x\,(\cos x - 1) = 0$

$\cos x = 0$, $\cos x = 1$.

On $[0, 2\pi)$...

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad$ $x = 0$

All solutions...

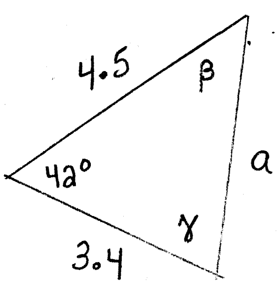
$x = \frac{\pi}{2} + 2\pi n$

$x = \frac{3\pi}{2} + 2\pi n$

$x = 2\pi n$

where n is any integer.

3. (10 points [8]) Given the triangle with $\alpha = 42°$, $b = 3.4$ meters, and $c = 4.5$ meters, find the remaining angles and side length. Round your final answers to the nearest tenth.



Law of Cosines...

$$a^2 = (4.5)^2 + (3.4)^2 - 2(4.5)(3.4)\cos 42°$$

$$a^2 \approx 9.069768 \Rightarrow \boxed{a \approx 3.0\text{ m}}$$

SAS ---

Not Ambiguous

Switch to Law of Sines...

$$\frac{\sin 42°}{a} = \frac{\sin \beta}{3.4} \Rightarrow \sin \beta \approx 0.7554$$

$$\boxed{\beta \approx 49.1°}$$

$$\gamma = 180° - (42° + \beta) \approx 88.9°$$

$$\boxed{\gamma \approx 88.9°}$$

Math 130 - Test 3B

November 18, 2020

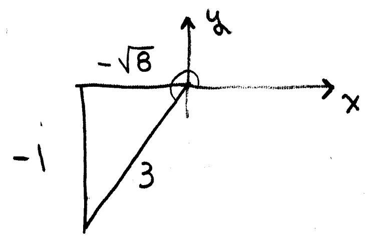
Name key

Score ________

Show all work to receive full credit. Supply explanations where necessary. When finding exact answers, simplify as much as possible. For each triangle described below, a is opposite α, b is opposite β, and c is opposite γ (unless otherwise indicated). This test is due November 30 by email. **You must work individually on this test.**

---

1. (8 points [3,6]) Given that x is a 3rd quadrant angle with $\sin x = -1/3$, find the exact values of $\sin(x/2)$ and $\cos(x/2)$. Do not use a calculator for this problem.



$a^2 + (-1)^2 = 3^2$

$a^2 = 8$

$\sin x = -\frac{1}{3}$

$\cos x = -\frac{\sqrt{8}}{3}$

X IN QUAD 3

$\Downarrow$

$180° < x < 270°$

$\Downarrow$

$90° < \frac{x}{2} < 135°$

$\Downarrow$

$\frac{x}{2}$ IN QUAD 2

$\Downarrow$

$\cos\frac{x}{2}$ NEG

$\sin\frac{x}{2}$ POS

$$\cos\frac{x}{2} = -\sqrt{\frac{1+\cos x}{2}} = -\sqrt{\frac{1-\sqrt{8}/3}{2}} = -\sqrt{\frac{3-\sqrt{8}}{6}}$$

$$\sin\frac{x}{2} = +\sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1+\sqrt{8}/3}{2}} = \sqrt{\frac{3+\sqrt{8}}{6}}$$

2. (3 points [3,6]) Use a product-to-sum formula to rewrite $\cos 3t \sin t$.

$$\cos 3t \sin t = \sin t \cos 3t = \frac{1}{2}\left[\sin(4t) + \sin(-2t)\right] = \frac{1}{2}\left[\sin 4t - \sin 2t\right]$$

3. (3 points [3,6]) Use a sum-to-product formula to rewrite $\sin 7\theta - \sin 3\theta$.

$$\sin 7\theta - \sin 3\theta = 2\cos\left(\frac{10\theta}{2}\right)\sin\left(\frac{4\theta}{2}\right)$$

$$= \boxed{2\cos 5\theta \sin 2\theta}$$

4. (8 points [9]) Find the exact solutions: $4\cos^2 x - 3 = 0$
(Find all solutions.)

$$\cos^2 x = \frac{3}{4}$$

$$\cos x = \pm\frac{\sqrt{3}}{2}$$

SOLUTIONS IN $[0, 2\pi)$

$$x = \frac{\pi}{6}, \frac{11\pi}{6}$$

$$x = \frac{5\pi}{6}, \frac{7\pi}{6}$$

ALL SOLUTIONS...

$$\frac{\pi}{6} + 2k\pi, \quad \frac{11\pi}{6} + 2k\pi,$$

$$\frac{5\pi}{6} + 2k\pi, \quad \frac{7\pi}{6} + 2k\pi$$

k IS ANY INTEGER.

5. (8 points [9]) Find the exact solutions: $2\sin^2 x - 5\sin x + 2 = 0$
(Find all solutions. Helpful hint: Factor the left-hand side.)

$$2\sin^2 x - 5\sin x + 2 = 0$$

$$(2\sin x - 1)(\sin x - 2) = 0$$

$$\sin x = \frac{1}{2} \qquad \sin x = 2$$

SOLUTIONS IN $[0, 2\pi)$...

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$\sin x = 2$ HAS NO SOLUTIONS

ALL SOLUTIONS...

$$\frac{\pi}{6} + 2k\pi,$$

$$\frac{5\pi}{6} + 2k\pi$$

k IS ANY INTEGER.

6. (10 points [7]) Solve the triangle. Round to the nearest tenth. If two solutions exist, find both.

LAW OF SINES

$$\alpha = 26.0°, \quad a = 10.0 \text{ centimeters}, \quad b = 18.0 \text{ centimeters}$$

$$\frac{\sin 26°}{10} = \frac{\sin \beta}{18} = \frac{\sin \gamma}{c}.$$

$$\sin \beta = \frac{18 \sin 26°}{10}.$$

$\beta \approx 52.1°$ or $127.9°$

CASE 1: $\beta \approx 52.1°$

$\gamma \approx 101.9°$

$$c = \frac{10 \sin \gamma}{\sin 26°} \approx 22.3 \text{ cm}$$

CASE 2: $\beta \approx 127.9°$

$\gamma \approx 26.1°$

$$c = \frac{10 \sin \gamma}{\sin 26°} \approx 10.0 \text{ cm}$$

7. (8 points [8]) Solve the triangle. Round to the nearest tenth. If two solutions exist, find both.

LAW OF COSINES

$$a = 1.2 \text{ feet}, \quad b = 2.0 \text{ feet}, \quad c = 1.5 \text{ feet}$$

α:
$$(1.2)^2 = (2.0)^2 + (1.5)^2 - 2(2.0)(1.5)\cos\alpha$$

$$\Rightarrow \cos\alpha = \frac{(1.2)^2 - (2.0)^2 - (1.5)^2}{-2(2.0)(1.5)}$$

$$\Rightarrow \boxed{\alpha \approx 36.7°}$$

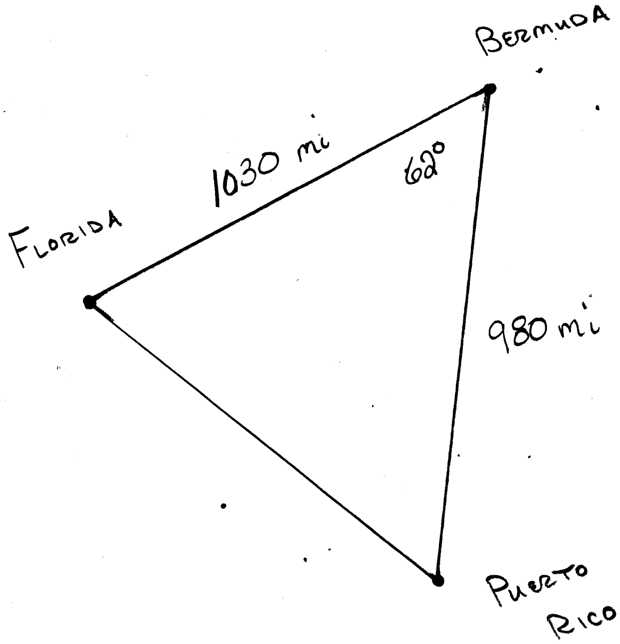
β:
$$(2.0)^2 = (1.2)^2 + (1.5)^2 - 2(1.2)(1.5)\cos\beta$$

$$\Rightarrow \cos\beta = \frac{(2.0)^2 - (1.2)^2 - (1.5)^2}{-2(1.2)(1.5)}$$

$$\boxed{\beta \approx 94.9°}$$

γ:
$$\boxed{\gamma = 180° - \alpha - \beta \approx 48.4°}$$

8. (7 points [7]) The Bermuda triangle is a region in the Atlantic Ocean that connects Bermuda, Florida, and Puerto Rico. The distance from Bermuda to Florida is 1030 miles, and the distance from Bermuda to Puerto Rico is 980 miles. The angle between those distances is 62°. Find the area of the Bermuda triangle. (Give units on your answer.)



$$\text{AREA} = \frac{bc \sin \alpha}{2}$$

$$\text{AREA} = \frac{(1030)(980) \sin 62^\circ}{2}$$

$$\approx 445{,}623.6 \text{ mi}^2$$

9. (5 points [9]) Find the exact solutions: $\tan 3x = 1$
(Find all solutions.)

LET $u = 3x$

$\tan u = 1$

$\Downarrow$

$u = \frac{\pi}{4}$

ALL u-SOLUTIONS ARE

$u = \frac{\pi}{4} + k\pi$, WHERE k IS ANY INTEGER

RESUBSTITUTE $u = 3x$ OR $x = \frac{u}{3}$ TO GET

x- SOLUTIONS ...

$x = \frac{\pi}{12} + \frac{k}{3}\pi$, k IS ANY INTEGER

10. (5 points [11]) Write as a complex number in standard form. Show all work.

(a) $2i(3+7i)^2$

$$2i\,(3+7i)(3+7i) = 2i\,(9+42i+49i^2)$$

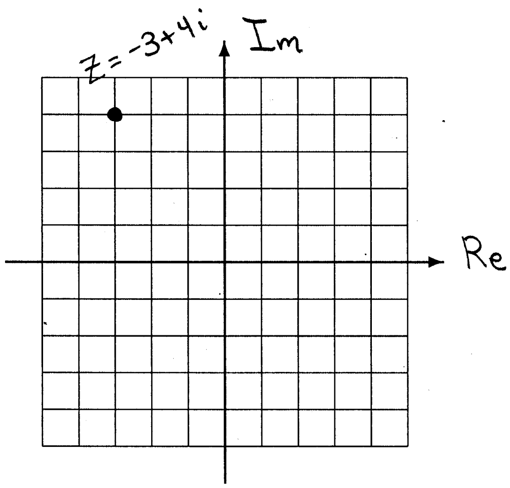
$$= 18i + 84i^2 + 98i^3$$

$$= 18i + (-84) - 98i = \boxed{-84-80i}$$

(b) $\frac{2-3i}{5-2i} \cdot \frac{5+2i}{5+2i} = \frac{(2-3i)(5+2i)}{25+4}$

$$= \frac{10-11i-6i^2}{29} = \boxed{\frac{16-11i}{29}}$$

11. (3 points [11,12]) Let $z = -3+4i$. Plot z in the complex plane. Then compute $|z|$.



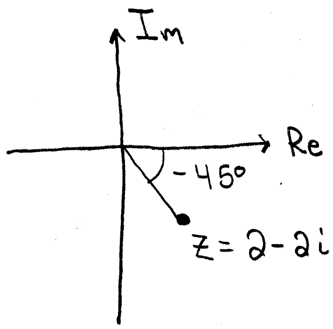
$$|z| = \sqrt{(-3)^2+(4)^2}$$

$$= \sqrt{9+16}$$

$$= \boxed{5}$$

12. (7 points [11]) Write each complex number in polar form. If necessary, round to the nearest tenth.

(a) $2 - 2i$



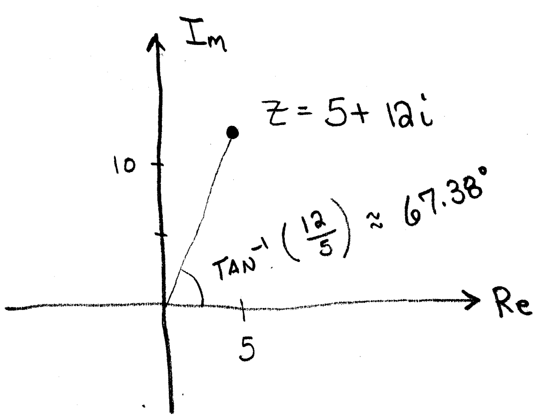
$r = |z| = \sqrt{(2)^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$

$\tan\theta = \frac{-2}{2} = -1$ $\qquad \tan^{-1}(-1) = -45°$

We'll use $\theta = 315°$

$$\boxed{z = 2\sqrt{2}\left(\cos 315° + i \sin 315°\right)}$$

(b) $5 + 12i$



$r = |z| = \sqrt{(5)^2 + (12)^2}$

$= \sqrt{169} = 13$

$$\boxed{z = 13\left(\cos 67.38° + i \sin 67.38°\right)}$$