Math 130 - Final Exam

December 16, 2020

Name key

Score ________

Show all work to receive full credit. For each triangle described below, a is opposite α, b is opposite β, and c is opposite γ (unless otherwise indicated). This test is due no later than December 17 at 11:59 pm.

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1. (5 points) Convert from radian measure to degree measure or vice versa. Give exact answers.

   (a) $\frac{7\pi}{15} \cdot \frac{180^\circ}{\pi} = \frac{7 \cdot 12^\circ}{1} = 84^\circ$

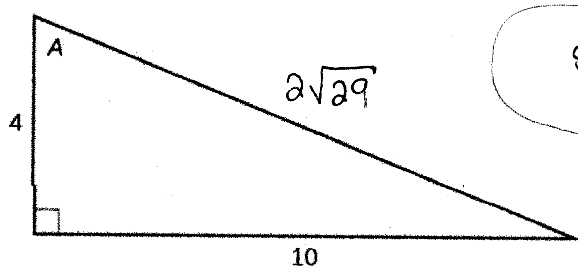
   (b) $288^\circ \cdot \frac{\pi}{180^\circ} = \frac{24\pi}{15} = \frac{8\pi}{5}$

2. (5 points) Refer to the right triangle shown below. Determine $\sin A$ and $\sec A$.

$\sqrt{10^2 + 4^2}$

$= \sqrt{116}$

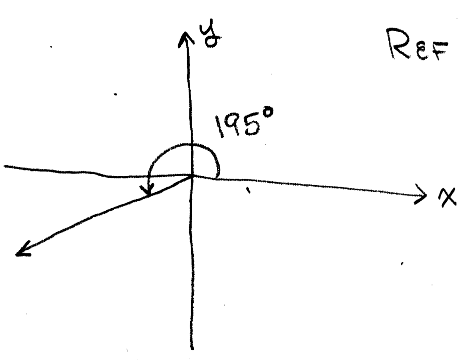
$= 2\sqrt{29}$



$\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{5}{\sqrt{29}}$

$\sec A = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{29}}{2}$

3. (5 points) Find the reference angle for 195°. Then explain how you would use the reference angle to determine sin 195°.



Ref $\angle = 195^\circ - 180^\circ = 15^\circ$

Ref. Angle $= 15^\circ$

$\sin 195^\circ = -\sin 15^\circ$

3rd Quad.

4. (5 points) Suppose that $\cos(\frac{\pi}{2} - \theta) = 0.625$. Use basic trig identities to find $\csc(-\theta)$.

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta = 0.625$$

$$\csc(-\theta) = \frac{1}{\sin(-\theta)} = \frac{1}{-\sin\theta} = -\frac{1}{0.625}$$

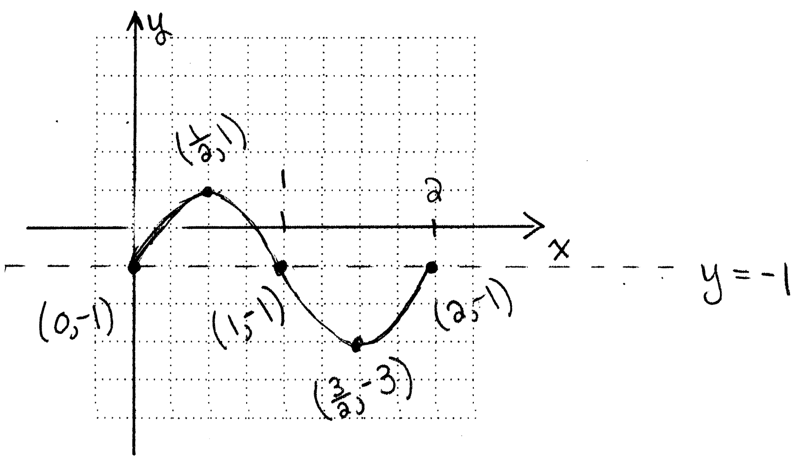
$$= \boxed{-1.6}$$

5. (5 points) Sketch exactly one full period of the graph of $y = -1 + 2\sin\pi x$. Label your axes and label the exact coordinates of three points on your graph.

AMPLITUDE = 2

MIDLINE: $y = -1$

PERIOD $= \frac{2\pi}{\pi} = 2$



6. (5 points) The angle β lies in the 3rd quadrant and $\sin\beta = -0.601815$. Find the angle β. Write your answer in degree measure rounded to the nearest tenth.

$$\sin^{-1}(-0.601815) \approx -37.0^\circ$$

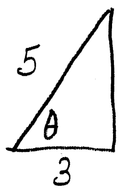
β IS IN QUAD 3 WITH REFERENCE $\angle = 37.0^\circ$.

$$\Downarrow$$

$$\boxed{\beta = 217.0^\circ}$$

7. (5 points) Use a right triangle to find the exact value of $\sin(\sec^{-1}(\frac{5}{3}))$.

$\theta = \sec^{-1}\left(\frac{5}{3}\right)$



$\sqrt{25-9}$
$= \sqrt{16} = 4$

$\sin\theta = \frac{4}{5}$

8. (5 points) Verify the idėntity: $\dfrac{1+\tan^2 x}{(1+\cot^2 x)\,\tan^2 x} = 1$

$$\frac{1+\tan^2 x}{(1+\cot^2 x)\tan^2 x} = \frac{\sec^2 x}{(1+\cot^2 x)\tan^2 x} = \frac{\sec^2 x}{\csc^2 x \tan^2 x}$$

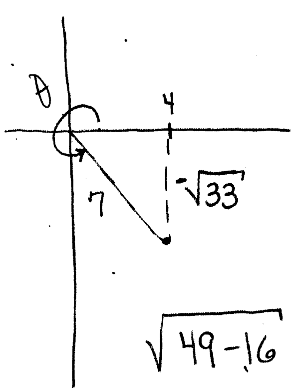
$$= \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}\cdot\frac{\sin^2 x}{\cos^2 x}} = \frac{\frac{1}{\cos^2 x}}{\frac{1}{\cos^2 x}} = 1 \checkmark$$

9. (5 points) Use the fact that $195 = 150 + 45$ to find the exact value of $\cos 195°$.

$$\cos(195°) = \cos(150° + 45°) = \cos 150° \cos 45° - \sin 150° \sin 45°$$

$$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{-\sqrt{6}-\sqrt{2}}{4}$$

10. (5 points) Given that θ is a 4th quadrant angle with $\cos\theta = 4/7$, find the exact value of $\sin 2\theta$.



$\sqrt{49-16}$
$= \sqrt{33}$

$$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(-\frac{\sqrt{33}}{7}\right)\left(\frac{4}{7}\right)$$

$\sin\theta = \frac{-\sqrt{33}}{7}$

$$= \frac{-8\sqrt{33}}{49}$$

11. (5 points) Find the exact solutions: $2\cos x + 1 = 0$
(Find all solutions.)

$\cos x = -\frac{1}{2}$

This happens twice in $[0, 2\pi)$:

$x = \frac{2\pi}{3}$ and $x = \frac{4\pi}{3}$.

All solutions: $x = \frac{2\pi}{3} + 2n\pi, \quad x = \frac{4\pi}{3} + 2n\pi$

n is any integer

12. (5 points) Find the exact solutions: $\sin^2 x + 6\sin x + 5 = 0$
(Find all solutions.)

$(\sin x + 5)(\sin x + 1) = 0$

$\underbrace{\sin x = -5}_{\text{Not possible}}$ $\quad$ $\underbrace{\sin x = -1}_{\text{Once in } [0, 2\pi)}$

$x = \frac{3\pi}{2}$

All solutions:

$x = \frac{3\pi}{2} + 2n\pi$

n is any integer

13. (5 points) Solve the triangle. Round your answers to the nearest tenth. If two solutions exist, find both.

$\alpha = 98.2°, \quad \beta = 56.3°, \quad b = 18.18$

Only one solution!

It has $\gamma = 180° - (98.2° + 56.3°)$

$= 25.5°$

$\gamma = 25.5°$

Law of sines...

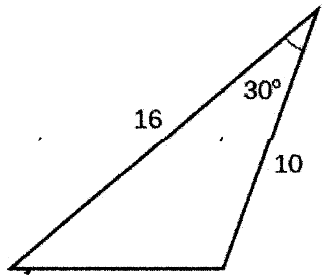
$\frac{\sin 98.2°}{a} = \frac{\sin 56.3°}{18.18}$

$a \approx 21.6$

$\frac{\sin 25.5°}{\gamma} = \frac{\sin 56.3°}{18.18}$

$\gamma \approx 9.4$

14. (5 points) Refer to the triangle below. Find the length of the unmarked side. Round your answer to the nearest tenth.



a

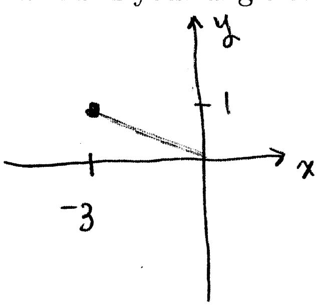
$$a^2 = 10^2 + 16^2 - 2(10)(16)\cos 30^\circ$$

$$= 356 - 320\left(\frac{\sqrt{3}}{2}\right) = 356 - 160\sqrt{3}$$

$$\approx 78.8719$$

$$\Rightarrow a \approx 8.9$$

15. (5 points) Write the complex number $z = -3 + i$ in polar form. Use degree measure and round your angle to the nearest tenth.



$$r = \sqrt{(-3)^2 + 1^2} = \sqrt{10}$$

$$\tan\theta = -\frac{1}{3} \qquad \tan^{-1}\left(-\frac{1}{3}\right) \approx -18.4^\circ$$

$$\Downarrow$$

$$\theta = 161.6^\circ$$

$$z = \sqrt{10}\,(\cos 161.6^\circ + i \sin 161.6^\circ)$$

16. (5 points) Let $z = 3(\cos 150^\circ + i \sin 150^\circ)$.

(a) Find z^5. Write your answer in polar form with your angle between 0° and 360°.

$$z^5 = 3^5(\cos 750^\circ + i \sin 750^\circ) \Rightarrow z = 243(\cos 30^\circ + i \sin 30^\circ)$$

(b) Write your answer from part (a) in standard form.

$$z = 243\left(\frac{\sqrt{3}}{2} + i\,\frac{1}{2}\right) = \frac{243\sqrt{3}}{2} + \frac{243}{2}i$$

17. (5 points) Without using a calculator, find the exact value of each. Explain your reasoning.

(a) $\log_2 \frac{1}{256}$ $= \log_2 2^{-8} = \boxed{-8}$ ← $y = \log_2 x$ AND $y = 2^x$ ARE INVERSES.

(b) $\ln e^{17/123}$ $= \boxed{\frac{17}{123}}$ ← $y = \ln x$ AND $y = e^x$ ARE INVERSES.

18. (5 points) Use properties of logarithms to completely expand: $\log\left[\frac{x^3(y+7)}{w\sqrt{z}}\right]$

$$\log\left[x^3(y+7)\right] - \log\left[wz^{1/2}\right]$$

$$= \log x^3 + \log(y+7) - \log w - \log z^{1/2}$$

$$= \boxed{3\log x + \log(y+7) - \log w - \tfrac{1}{2}\log z}$$

19. (5 points) Solve for x, and then use a calculator to check your work. Show your check step.

$$\log(4) + \log(-5x) = 2$$

$\log(-20x) = 2$ $\boxed{x = -5}$

$-20x = 10^2$

$-20x = 100$

$\log(4) + \log(25) = 2$ ✓

$0.60205... + 1.39794... = 2$

20. (5 points) Solve for x. Find the exact solution (which will contain a logarithm), and then round your answer to the nearest hundredth.

$$10e^{8x+3} + 2 = 8$$

$10e^{8x+3} = 6$

$e^{8x+3} = \frac{6}{10} = \frac{3}{5}$

$8x + 3 = \ln(3/5)$

$8x = \ln(3/5) - 3$

$x = \frac{\ln(3/5) - 3}{8}$

≈ -0.44