Math 131 - Quiz 3

September 16, 2020

Name key

Score

Show all work to receive full credit. Supply explanations when necessary. This quiz is due on September 21.

1. (2 points) Find the limit analytically: $\lim_{x \to 0} \frac{2 \tan 3x}{9x}$ 0/0 MORE WORK

$$\lim_{x \to 0} \frac{2 \sin 3x}{9x \cos 3x} = \lim_{x \to 0} \left[\left(\frac{\sin 3x}{3x} \right) \left(\frac{2}{3 \cos 3x} \right) \right]$$

$$= (1)\left(\frac{2}{3}\right) = \frac{2}{3}$$

2. (2 points) Compute the limit, $\lim_{x \to 1^+} f(x)$, or explain why it fails to exist.

$$f(x) = \begin{cases} x \sin \pi x, & x \le 1 \\ 2x + \tan x, & x > 1 \end{cases}$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (2x + \tan x)$$

$$= 2 + \tan 1 \approx 3.558$$

3. (2 points) Find the constant k so that g is continuous everywhere. (Your work should address the three things we check when testing for continuity.)

$$g(x) = \begin{cases} x^2 - 8x + 15, & x < 4 \\ k - x \cos \pi x, & x \ge 4 \end{cases}$$

RIGHT NOW, g IS CONTINUOUS EVERYWHERE WITH THE POSSIBLE EXCEPTION OF $X = 4$. LET'S WORK ON $X = 4$...

$$\lim_{x \to 4^-} g(x) = \lim_{x \to 4} (x^2 - 8x + 15) = -1$$

$$\lim_{x \to 4^+} g(x) = \lim_{x \to 4} (k - x \cos \pi x) = k - 4 = g(4)$$

$$k - 4 = -1 \Rightarrow k = 3$$

4. (2 points) Find and classify the discontinuities of the function $f(x) = \dfrac{x^2 - 3x + 2}{x^2 - 4}$.

f IS DISCONTINUOUS WHEN $x^2 - 4 = 0$,

THAT IS, AT $x = \pm 2$

$$f(x) = \frac{\cancel{(x-2)}(x-1)}{\cancel{(x-2)}(x+2)}$$

$x = 2$ IS A REMOVABLE DISCONT

SINCE $\lim_{x \to 2} f(x) = \frac{1}{4}$

$x = -2$ IS AN INFINITE DISCONT.

SINCE f HAS THE FORM $\frac{-3}{0}$ AT $x = -2$.

5. (2 points) Determine the limit analytically. You may need to use $+\infty$, $-\infty$, or DNE.

$$\lim_{y \to 5^+} \left(\frac{2y - 10}{y^2 - 10y + 25} \right)$$

$\frac{2}{0}$ SOME KIND OF ∞

$$\lim_{y \to 5^+} \frac{2(y-5)}{(y-5)(y-5)} = \lim_{y \to 5^+} \frac{2}{y-5}$$

$$= \boxed{+\infty}$$

SINCE TO THE RIGHT OF $y = 5$ THE FRACTION IS $\frac{+}{+} = +$.