Math 131 - Quiz 6

October 21, 2020

Name key

Score ______

Show all work to receive full credit. Supply explanations when necessary. You must work individually on this quiz. This quiz is due no later than October 26.

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1. (3 points) Consider the equation $x^2 + 2xy - 3y^2 = -7$ and its graph.

   (a) Use implicit differentiation to find dy/dx.

$$\frac{d}{dx}\left(x^2 + 2xy - 3y^2\right) = \frac{d}{dx}(-7)$$

$$2x + 2x\frac{dy}{dx} + 2y - 6y\frac{dy}{dx} = 0$$

$$(2x - 6y)\frac{dy}{dx} = -2x - 2y$$

$$\boxed{\frac{dy}{dx} = \frac{-2x - 2y}{2x - 6y}}$$

   (b) Find an equation of the line tangent to the graph the point $(x, y) = (1, 2)$.

$$m = \left.\frac{dy}{dx}\right|_{(x,y)=(1,2)} = \frac{-6}{-10} = \frac{3}{5}$$

$$y - 2 = \frac{3}{5}(x - 1)$$

or

$$y = \frac{3}{5}x + \frac{7}{5}$$

   (c) Find an equation of the line normal to the graph at the point $(x, y) = (1, 2)$.

$$m_{\perp} = -\frac{5}{3}$$

$$y - 2 = -\frac{5}{3}(x - 1)$$

or

$$y = -\frac{5}{3}x + \frac{11}{3}$$

2. (2 points) Let $f(x) = x + \sqrt{x}$.. Find $(f^{-1})'(2)$.

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$$f'(x) = 1 + \frac{1}{2}x^{-1/2}$$

$$f^{-1}(2) = u \iff f(u) = 2 \Rightarrow x + \sqrt{x} = 2 \Rightarrow x = 1$$

$$(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(1)} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

3. (2 points) Find an equation of the line tangent to the graph of $y = \sin^{-1} 4x$ at the point where $x = -1/8$.

POINT: $X = -1/8$, $y = \sin^{-1}(-4/8)$
$= -\frac{\pi}{6}$.

$\left(-\frac{1}{8}, -\frac{\pi}{6}\right)$

SLOPE: $\frac{dy}{dx} = \frac{4}{\sqrt{1-(4x)^2}}$

$m = \left.\frac{dy}{dx}\right|_{x=-\frac{1}{8}} = \frac{4}{\sqrt{3/4}} = \frac{8}{\sqrt{3}}$

TAN. LINE:

$$\boxed{y + \frac{\pi}{6} = \frac{8}{\sqrt{3}}\left(x + \frac{1}{8}\right)}$$

4. (3 points) Determine each derivative. Do not simplify, but show all your work.

(a) $\frac{d}{dx}\left[x \cot^{-1}(x^2)\right]$

$$= \cot^{-1}(x^2) + x\left(\frac{-2x}{1+x^4}\right) = \boxed{\cot^{-1}(x^2) - \frac{2x^2}{1+x^4}}$$

(b) $\frac{d}{dt} e^{t^3 \ln t}$ $= e^{t^3 \ln t} \frac{d}{dt}\left(t^3 \ln t\right)$

$$= \boxed{e^{t^3 \ln t}\left[3t^2 \ln t + t^2\right]}$$

(c) $\frac{d}{dr} \frac{\ln(9r)}{2^r}$ $= \frac{2^r\left(\frac{9}{9r}\right) - \ln(9r)\, 2^r \ln 2}{2^{2r}} = \boxed{\frac{\frac{1}{r} - \ln(9r) \ln 2}{2^r}}$