

Math 131 - Quiz 7

October 28, 2020

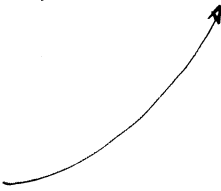
Name key

Score _____

The following problems are from the suggested homework. Show all work to receive full credit. Supply explanations when necessary. You must work individually on this quiz. This quiz is due November 4.

1. (2 points) Let $f(x) = x - \frac{2}{x}$. Compute $(f^{-1})'(1)$.

$$(f^{-1})'(1) = \frac{1}{f'(f^{-1}(1))} = \frac{1}{f'(2)} = \frac{1}{1 + \frac{2}{4}} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

$$\begin{aligned} f^{-1}(1) &= w \\ \Downarrow \\ f(w) &= 1 \\ \Downarrow \\ w - \frac{2}{w} &= 1 \\ \Downarrow \\ w &= 2 \quad (\text{By OBSERVATION}) \end{aligned}$$
$$f'(x) = 1 + \frac{2}{x^2}$$


2. (2 points) Determine $\frac{dy}{dx}$ if $y = (1 + \tan^{-1} x)^3$.

$$\frac{dy}{dx} = 3(1 + \tan^{-1} x)^2 \left(\frac{1}{1+x^2} \right)$$

3. (2 points) Find an equation of the line tangent to the graph of $x^2y^2 + 5xy = 14$ at the point (2, 1).

$$\frac{d}{dx}(x^2y^2 + 5xy) = 0$$

$$2xy^2 + x^2 \cdot 2y \frac{dy}{dx} + 5y + 5x \frac{dy}{dx} = 0$$

$$(2x^2y + 5x) \frac{dy}{dx} = -2xy^2 - 5y$$

$$\frac{dy}{dx} = \frac{-2xy^2 - 5y}{2x^2y + 5x}$$

$$m = \left. \frac{dy}{dx} \right|_{(2,1)} = \frac{-4-5}{8+10} = -\frac{9}{18} = -\frac{1}{2}$$

$$y - 1 = -\frac{1}{2}(x - 2)$$

$$\text{or } y = -\frac{1}{2}x + 2$$

4. (2 points) Let $f(x) = \log_7(6x^4 + 3)^5$. Find $f'(x)$.

$$f(x) = \frac{\ln(6x^4 + 3)^5}{\ln 7} = \frac{5}{\ln 7} \ln(6x^4 + 3)$$

$$f'(x) = \frac{5}{\ln 7} \left(\frac{24x^3}{6x^4 + 3} \right)$$

5. (2 points) Use logarithmic differentiation to find $\frac{dy}{dx}$ if $y = \frac{x+11}{\sqrt[3]{x^2-4}}$.

$$\ln y = \ln \left(\frac{x+11}{\sqrt[3]{x^2-4}} \right) = \ln(x+11) - \frac{1}{3} \ln(x^2-4)$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{x+11} - \frac{2x}{3(x^2-4)}$$

$$\frac{dy}{dx} = \left(\frac{x+11}{\sqrt[3]{x^2-4}} \right) \left(\frac{1}{x+11} - \frac{2x}{3(x^2-4)} \right)$$