Math 131 - Quiz 9

December 2, 2020

Name key

Score ________

The following problems are from the suggested homework. Show all work to receive full credit. Supply explanations when necessary. You must work individually on this quiz. This quiz is due by email on December 7.

1. (2 points) Evaluate the limit: $\lim_{x\to-\infty} \frac{\sqrt{4x^2-1}}{x+2}$

For $x < 0$, $\frac{1}{\sqrt{x^2}} = -\frac{1}{x}$.

$$\lim_{x\to-\infty} \frac{\sqrt{4x^2-1} \cdot 1/\sqrt{x^2}}{x+2 \cdot 1/-x} = \lim_{x\to-\infty} \frac{\sqrt{4-\frac{1}{x^2}}}{-1-\frac{2}{x}} = \frac{\sqrt{4}}{-1}$$

$$= \boxed{-2}$$

2. (2 points) Find the vertical and horizontal asymptotes of the graph of $f(x) = \frac{x\sin x}{x^2-1}$.

$$f(x) = \frac{x \sin x}{(x+1)(x-1)}$$

$\boxed{\text{V.A. } x=-1,\ x=1}$ (Denom $= 0$, but numer $\neq 0$)

H.A... $\lim_{x\to\pm\infty} \frac{x\sin x}{x^2-1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\pm\infty} \frac{\frac{\sin x}{x}}{1-\frac{1}{x^2}} = \frac{0}{1} = 0$

$\boxed{\text{H.A. } y=0}$

$0/0$

3. (2 points) Evaluate the limit: $\lim_{x \to 0} \frac{e^x - x - 1}{x^2}$.

L' Hôpital's Rule twice.

$$= \lim_{x \to 0} \frac{e^x - 1}{2x} \quad 0/0$$

$$= \lim_{x \to 0} \frac{e^x}{2} = \boxed{\frac{1}{2}}$$

$\infty \cdot 0$

4. (2 points) Evaluate the limit: $\lim_{x \to \infty} x \sin\left(\frac{1}{x}\right)$

$$\lim_{x \to \infty} \frac{\sin \frac{1}{x}}{\frac{1}{x}} = \lim_{x \to \infty} \frac{\left(\cos \frac{1}{x}\right)\cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{\left(-\frac{1}{x^2}\right)}}$$

L' Hopital's Rule

$$= \lim_{x \to \infty} \cos \frac{1}{x} = \cos 0 = \boxed{1}$$

5. (2 points) Find $f(x)$ if $f'(x) = \frac{2}{x^2} - \frac{x^2}{2}$ and $f(1) = 0$.

$$f(x) = \int \left(2x^{-2} - \frac{1}{2}x^2\right) dx = -2x^{-1} - \frac{1}{6}x^3 + C$$

$$= -\frac{2}{x} - \frac{x^3}{6} + C$$

$$f(1) = 0 = -\frac{2}{1} - \frac{1}{6} + C$$

$$C = 2 + \frac{1}{6} = \frac{13}{6}$$

$$\boxed{f(x) = -\frac{2}{x} - \frac{x^3}{6} + \frac{13}{6}}$$