Math 131 - Test 1
September 23, 2020

Name key

Score ______

Show all work to receive full credit. Supply explanations where necessary. When evaluating limits, you may need to use $+\infty$, $-\infty$, or DNE (does not exist).

1. (6 points) Use a table of numerical values to approximate the following limit. Your table must show function values at six or more points.

$$\lim_{x\to 1} \frac{1-2^{x-1}}{5x-5}$$

Let $f(x) = \dfrac{1-2^{x-1}}{5x-5}$

x	$f(x)$
0.9	-0.133934
1.1	-0.143547
0.99	-0.138150
1.01	-0.139111
0.999	-0.138581
1.001	-0.138677

It looks like

$$\lim_{x\to 1} \frac{1-2^{x-1}}{5x-5} \approx -0.139$$

2. (8 points) Let $f(x) = \dfrac{x^2-2x-3}{|x-3|}$.

$$f(x) = \frac{(x-3)(x+1)}{|x-3|}$$

(a) Compute the limit: $\displaystyle\lim_{x\to 3^+} f(x)$

$$\lim_{x\to 3^+} f(x) = \underbrace{\lim_{x\to 3^+} \frac{(x-3)(x+1)}{x-3}}_{\text{For } x>3,\ |x-3| = x-3} = \lim_{x\to 3} (x+1) = \boxed{4}$$

(b) Compute the limit: $\displaystyle\lim_{x\to 3^-} f(x)$

$$\underbrace{\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{(x-3)(x+1)}{-(x-3)}}_{\text{For } x<3,\ |x-3| = -(x-3)} = \lim_{x\to 3} -(x+1) = \boxed{-4}$$

(c) What do the results of parts (a) and (b) tell you about $\displaystyle\lim_{x\to 3} f(x)$?

Limit from right $\neq$ Limit from left

$\Rightarrow$ $\boxed{\text{Limit DNE.}}$

EACH OF THESE IS A $\frac{5}{0}$ FORM. THERE IS SOME KIND OF INFINITE LIMIT FOR EACH.

3. (16 points) In each problem below, determine whether the limit is $+\infty$, $-\infty$, or DNE. Show work or explain your reasoning.

(a) $\lim_{x \to 5^+} \frac{x}{x-5}$

FOR $X \to 5^+$, NUMERATOR IS POS. & DENOM. IS POS.

$\frac{+}{+} = + \Rightarrow \boxed{\text{LIMIT IS } +\infty.}$

(b) $\lim_{x \to 5^-} \frac{x}{x-5}$

FOR $X \to 5^-$, NUMERATOR IS POS. & DENOM. IS NEG.

$\frac{+}{-} = - \Rightarrow \boxed{\text{LIMIT IS } -\infty.}$

(c) $\lim_{x \to 5} \frac{x}{x-5}$

$\boxed{\text{LIMIT DNE}}$ BASED ON (a) & (b)

(d) $\lim_{x \to 5} \frac{x}{(x-5)^2}$

FOR $X \to 5$ FROM EITHER SIDE, NUMERATOR IS POS. & DENOM. IS POS.

$\frac{+}{+} = + \Rightarrow \boxed{\text{LIMIT IS } +\infty.}$

4. (8 points) Discuss the continuity of g. At which points is g continuous/discontinuous, and how do you know?

$$g(x) = \begin{cases} 6x + \sin(\pi x), & x \leq -1 \\ x^2 - 8x - 15, & -1 < x \leq 3 \\ x\cos(\pi x) - 9x, & x > 3 \end{cases}$$

EACH "PIECE" IS CONTINUOUS EVERYWHERE ON ITS RESPECTIVE DOMAIN.

WE NEED ONLY CHECK WHERE THE PIECES "CONNECT":

$X = -1$ AND $X = 3$

① $g(-1) = -6$

② $\lim_{X \to -1^-} g(x) = \lim_{X \to -1^-} 6x + \sin(\pi x) = -6$

③ $\lim_{X \to -1^+} g(x) = \lim_{X \to -1^+} x^2 - 8x - 15 = -6$

④ g IS CONT AT $X = -1$

① $g(3) = 9 - 24 - 15 = -30$

② $\lim_{X \to 3^-} g(x) = \lim_{X \to 3^-} x^2 - 8x - 15 = -30$

③ $\lim_{X \to 3^+} g(x) = \lim_{X \to 3^+} x\cos(\pi x) - 9x = -3 - 27 = -30$

④ g IS CONT AT $X = 3$

$\boxed{g \text{ IS CONTINUOUS EVERYWHERE.}}$

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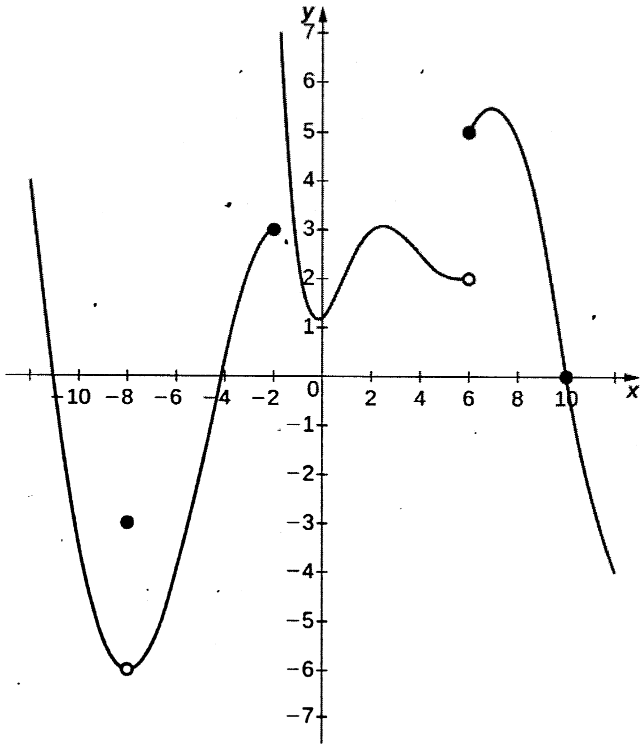
5. (10 points) Referring to the graph of $y = f(x)$ shown below, determine each of the following or explain why it does not exist.



(a) $\lim_{x \to 10} f(x)$ = 0

(b) $f(-2)$ = 3

(c) $\lim_{x \to -8} f(x)$ = -6

(d) $\lim_{x \to -2+} f(x)$ = $+\infty$

(e) $\lim_{x \to 6-} f(x)$ = 2

6. (6 points) Refer to the function $y = f(x)$ whose graph is shown above. Classify the discontinuities of f.

REMOVABLE DISCONT. AT X = -8

INFINITE DISCONT. AT X = -2

JUMP DISCONT. AT X = 6

7. (24 points) Determine each limit analytically, or explain why the limit does not exist. You may need to use $+\infty$, $-\infty$, or DNE.

(a) $\lim_{x\to 0} \frac{\tan \pi x}{5x}$ % MORE WORK.

$$= \lim_{x\to 0}\left(\frac{\sin \pi x}{5x}\cdot\frac{1}{\cos \pi x}\right) = \lim_{x\to 0}\left(\frac{\pi}{5}\cdot\frac{\sin \pi x}{\pi x}\cdot\frac{1}{\cos \pi x}\right)$$

$$= \frac{\pi}{5}\cdot(1)\cdot(1) = \boxed{\frac{\pi}{5}}$$

(b) $\lim_{r\to 3^-}\left(\frac{r^2-2r-3}{r^2-9}\right)$ % MORE WORK

$$\lim_{r\to 3^-}\frac{(r-3)(r+1)}{(r-3)(r+3)} = \lim_{r\to 3^-}\frac{r+1}{r+3} = \frac{4}{6} = \boxed{\frac{2}{3}}$$

% MORE WORK

(c) $\lim_{x\to 1}\frac{1-\sqrt{x}}{1-x}$

$$\lim_{x\to 1}\frac{1-\sqrt{x}}{1-x}\cdot\frac{1+\sqrt{x}}{1+\sqrt{x}} = \lim_{x\to 1}\frac{\cancel{1-x}}{\cancel{(1-x)}(1+\sqrt{x})} = \lim_{x\to 1}\frac{1}{1+\sqrt{x}}$$

$$= \boxed{\frac{1}{2}}$$

(d) $\lim_{t\to 0}\frac{(t+5)^2-25}{t}$ % MORE WORK

$$\lim_{t\to 0}\frac{t^2+10t}{t} = \lim_{t\to 0}(t+10) = \boxed{10}$$

8. (6 points) Determine all vertical asymptotes of the graph of $h(x) = \dfrac{x^2+2x-8}{x^2-4}$.
(You can use your graphing calculator for help, but you must show computational work for full credit.)

$h(x) = \dfrac{(x+4)(x-2)}{(x+2)(x-2)}$

ZEROS OF DENOM ARE $X=-2$, $X=2$.

THESE ARE THE POSSIBLE ASYMPTOTES.

LET'S CHECK...

$\lim\limits_{x\to 2} h(x) \overset{0/0}{=} \lim\limits_{x\to 2} \dfrac{x+4}{x+2}$

$= \dfrac{6}{4} \Rightarrow X=2$ IS NOT A VA.

$\lim\limits_{x\to -2} h(x)$ HAS $\dfrac{\text{NONZERO}}{\text{ZERO}}$ FORM.

$\Rightarrow$ $X=-2$ IS A VA.

9. (6 points) Consider the following table of values for the function f.

x	-0.1	0.1	-0.01	0.01	-0.001	0.001
$f(x)$	2.8	3.2	2.98	3.02	2.998	3.002

(a) Given the table of values and no other information, is it possible that $\lim\limits_{x\to 0} f(x)$ does not exist? Explain.

YES, WE HAVE NO IDEA WHAT f IS DOING AS WE GET EVEN CLOSER TO $X=0$.
IT LOOKS LIKE $f(x) \to 3$ BUT WE REALLY DON'T HAVE ENOUGH INFO.

(b) Given the table of values and the fact that f is continuous everywhere, is it possible that $\lim\limits_{x\to 0} f(x)$ does not exist? Explain.

NO, IF f IS CONTINUOUS EVERYWHERE, THEN IT HAS A LIMIT EVERYWHERE.
(HOWEVER, WE CAN'T SAY FOR CERTAIN WHAT THE LIMIT IS.)

(c) Given the table of values and the fact that f has a removable discontinuity at $x=0$, is it possible that $\lim\limits_{x\to 0} f(x)$ does not exist? Explain.

NO, THE LIMIT MUST EXIST FOR A DISCONTINUITY TO BE REMOVABLE.

10. (10 points) Determine whether each statement is true (T) or false (F).

(a) F If f is defined at $x = c$, then $\lim_{x \to c} f(x)$ exists.

(b) F $\lim_{x \to 0} \sqrt{x} = 0$

(c) F If the graph of f has the vertical asymptote $x = 7$, then $f(7)$ is not defined.

(d) T If $\lim_{x \to 2} f(x) = f(2)$, then f is continuous at $x = 2$.

(e) T The limit of a polynomial function can always be found by direct substitution.

(f) T If $\lim_{x \to 3} g(x) = 7$, then $\lim_{x \to 3^+} g(x) = 7$.

(g) F A jump discontinuity might also be a removable discontinuity.

(h) F If $h(-2) = 0$, then $\lim_{x \to -2} h(x) = 0$.

(i) F If f and g are polynomials and $g(3) = 0$, then the graph of $\dfrac{f(x)}{g(x)}$ must have a vertical asymptote at $x = 3$.

(j) F The limit of a rational function can always be found by direct substitution.