Math 131 - Test 2

October 14, 2020

Name key

Score ________

Show all work to receive full credit. Supply explanations where necessary. Unless otherwise indicated, use differentiation rules for all derivatives, and do not simplify.

1. (8 points) Let $f(x) = 3x^2 - x$. Use the limit definition of derivative to determine $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[3(x+h)^2 - (x+h)] - [3x^2 - x]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{x} - h + \cancel{3x^2} + \cancel{x}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(6x + 3h - 1)}{\cancel{h}} = \boxed{6x - 1}$$

2. (4 points) Use differentiation rules to confirm your result above. Then find an equation of the line tangent to the graph of $f(x) = 3x^2 - x$ at the point where $x = -2$.

$f'(x) = 6x - 1$

$m = f'(-2) = -13$

POINT: $x = -2$

$y = f(-2) = 14$

$(-2, 14)$

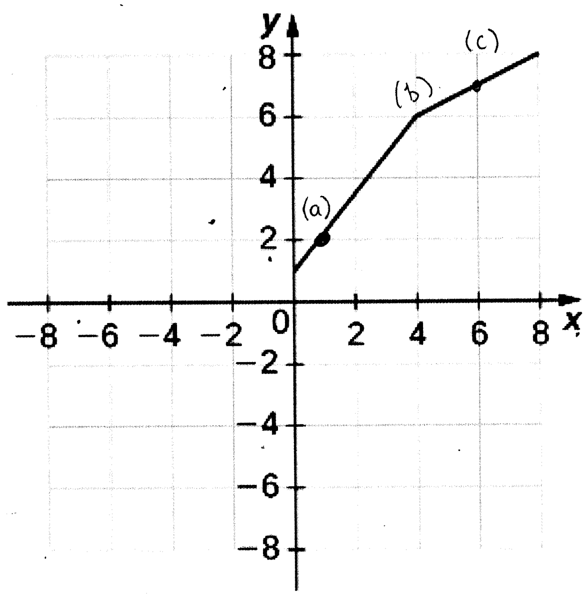
TANGENT LINE:

$y - 14 = -13(x + 2)$

OR

$y = -13x - 12$

3. (6 points) The graph of the function f is shown below. Use the graph to determine each of the following. Show work or explain.



(a) $f'(1)$ $= \dfrac{\Delta y}{\Delta x} = \dfrac{6-1}{4-0} = \boxed{\dfrac{5}{4}}$

(b) $f'(4)$ $\boxed{\text{DNE}}$ (SHARP POINT)

(c) $f'(6)$ $= \dfrac{\Delta y}{\Delta x} = \dfrac{8-6}{8-4} = \dfrac{2}{4} = \boxed{\dfrac{1}{2}}$

4. (8 points) State two ways in which a function's derivative may fail to exist at a point where the function itself is defined. Then use a single graph to illustrate both ways. Label which is which.

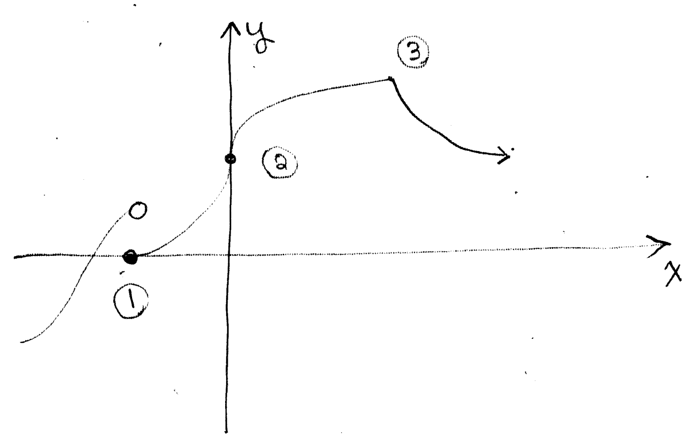
I'LL GIVE THREE WAYS...

① DISCONTINUITY AT A POINT

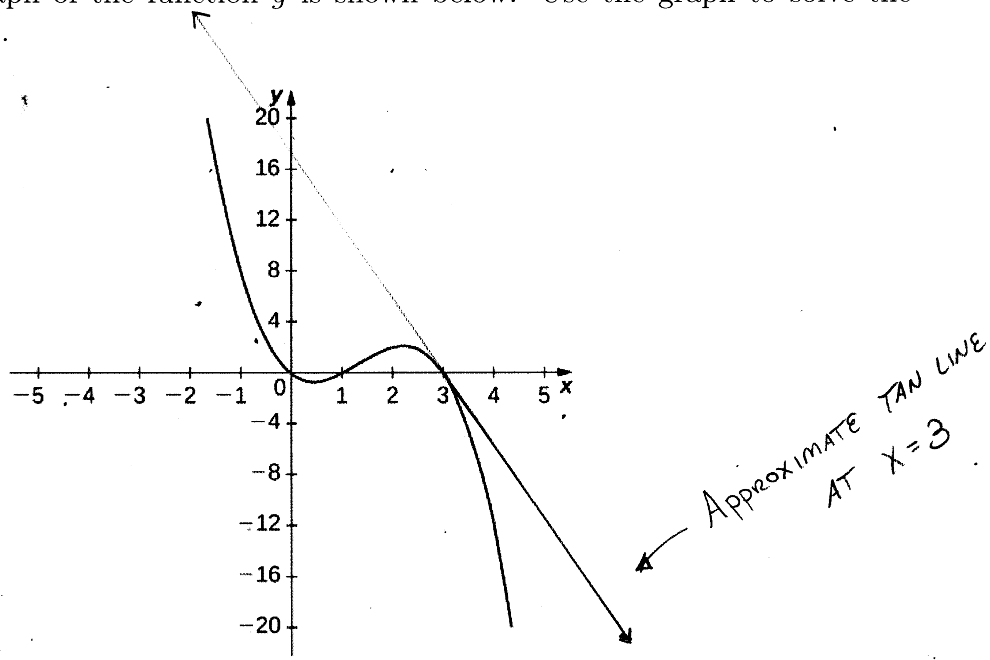
② VERTICAL TANGENT LINE AT POINT

③ SHARP POINT ON GRAPH AT POINT

(SLOPE FROM LEFT ≠ SLOPE FROM RIGHT)



5. (8 points) The graph of the function g is shown below. Use the graph to solve the following problems.



(a) Sketch the tangent line at $x = 3$. Then use your tangent line to estimate $g'(3)$.

Using $(0,17)$ and $(3,0)$ $\quad g'(3) \approx \dfrac{17-0}{0-3} = -\dfrac{17}{3}$ $\quad \boxed{g'(3) \approx -\dfrac{17}{3}}$

(b) Find approximate coordinates of a point on the graph at which $g'(x) = 0$. Explain how you know that $g'(x) = 0$ at your point.

THERE IS A HORIZONTAL TANGENT LINE AT ABOUT $\boxed{\left(\frac{1}{2}, -1\right)}$

$m = 0$

6. (5 points) Use the quotient rule to derive the formula for the derivative of $y = \csc x$.

$$\frac{d}{dx} \csc x = \frac{d}{dx} \frac{1}{\sin x} = \frac{(\sin x)(0) - (1)(\cos x)}{\sin^2 x}$$

$$= \frac{-\cos x}{\sin^2 x} = -\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}$$

$$= \boxed{-\cot x \csc x}$$

7. A potato is launched vertically upward with a velocity of 100 ft/sec from over the edge of the top of an 85-ft building. Use the position function

$$s(t) = -16t^2 + v_0 t + s_0,$$

where s represents height (in feet) at time t (in seconds), to solve the following problems.

(a) (2 points) Determine the function $s(t)$ that gives the potato's height at time t.

$$s(t) = -16t^2 + 100t + 85$$

(b) (2 points) Determine the average rate of change the potato's height over the interval from $t = 0$ to $t = 3$. (Include units with your answer.)

$$\frac{\Delta s}{\Delta t} = \frac{s(3) - s(0)}{3 - 0} = \frac{241 - 85}{3} = 52 \text{ FT}/\text{s}$$

(c) (2 points) Determine the function that gives the potato's velocity at time t.

$$v(t) = s'(t) = -32t + 100$$

(d) (2 points) Determine the potato's velocity after 4 seconds. (Include units with your answer.)

$$v(4) = -32(4) + 100 = -28 \text{ FT}/\text{s}$$

(e) (2 points) What is the acceleration of the potato? (Include units with your answer.)

$$a(t) = v'(t) = -32 \text{ FT}/\text{s}^2.$$

(f) (4 points) Determine the potato's maximum height. (Include units with your answer.)

$$v(t) = 0 \Rightarrow -32t + 100 = 0$$

$$t = \frac{100}{32} = 3.125$$

$$s(3.125) = 241.25 \text{ FT}$$

(g) (3 points) When does the potato hit the ground?

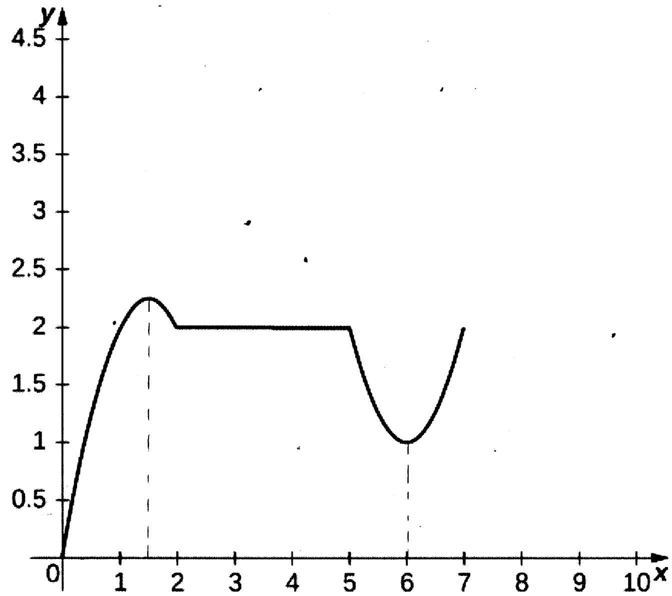
$$s(t) = 0$$

$$-16t^2 + 100t + 85 = 0$$

$$t = \frac{-100 - \sqrt{100^2 + 4(16)(85)}}{-32}$$

$$\approx 7.008 \text{ s}$$

8. (6 points) The graph below shows the position $y = s(x)$ of an object moving along a line. Use the graph to answer the following questions.



(a) When is the object stopped?

$S'(x) = 0$ WHEN $= 1.5,\ = 6,$ AND IN $[2, 5]$

(b) When is the object moving backward?

$S'(x) < 0$ WHEN IN $(1.5, 2)$ AND IN $(5, 6)$

9. (6 points) Determine the higher-order derivative: $\dfrac{d^3}{dx^3}\left(2x^8 - 5x^4 - 5\sin x\right)$

$$\frac{dy}{dx} = 16x^7 - 20x^3 - 5\cos x$$

$$\frac{d^2y}{dx^2} = 112x^6 - 60x^2 + 5\sin x$$

$$\boxed{\frac{d^3y}{dx^3} = 672x^5 - 120x + 5\cos x}$$

10. (12 points) The table below gives the values of the functions f and g and their derivatives at selected values of x.

x	0	1	2
$f(x)$	3	-5	0
$f'(x)$	1	-1	-4
$g(x)$	-2	2	1
$g'(x)$	2	6	-7

(a) If $h(x) = 3f(x) + 2g(x) + x$, compute $h'(2)$.

$$h'(x) = 3f'(x) + 2g'(x) + 1$$

$$h'(2) = 3(-4) + 2(-7) + 1 = \boxed{-25}$$

(b) If $h(x) = f(x) \cdot g(x)$, compute $h'(0)$.

$$h'(x) = f'(x)g(x) + f(x)g'(x)$$

$$h'(0) = 1(-2) + 3(2) = \boxed{4}$$

(c) If $h(x) = \dfrac{g(x)}{f(x)}$, compute $h'(1)$.

$$h'(x) = \frac{f(x)g'(x) - f'(x)g(x)}{[f(x)]^2}$$

$$h'(1) = \frac{-5(6) - (-1)(2)}{(-5)^2} = \boxed{\frac{-28}{25}}$$

(d) If $h(x) = f(g(x))$, compute $h'(2)$.

$$h'(x) = f'(g(x))\, g'(x)$$

$$h'(2) = f'(1)\, g'(2) = (-1)(-7) = \boxed{7}$$

11. (20 points) Differentiate. Do not simplify.

(a) $\dfrac{d}{dw}\left(\dfrac{w^5 - 2w^3 + 8}{w^3}\right) = \dfrac{d}{dw}\left(w^2 - 2 + 8w^{-3}\right)$

$= 2w - 24w^{-4} = \boxed{2w - \dfrac{24}{w^4}}$

POWER RULE

(b) $\dfrac{d}{d\theta}(\theta \sec\theta) = \boxed{\theta \sec\theta \tan\theta + \sec\theta}$

PRODUCT RULE

(c) $\dfrac{d}{dx}\tan(\sqrt{x}) = \sec^2(\sqrt{x})\dfrac{d}{dx}\sqrt{x} = \sec^2(\sqrt{x})\left(\frac{1}{2}x^{-1/2}\right)$

$= \boxed{\dfrac{\sec^2\sqrt{x}}{2\sqrt{x}}}$

CHAIN RULE

(d) $\dfrac{d}{dt}\left[(t^2+1)^3(\sin t)^2\right] = (\sin t)^2 \dfrac{d}{dt}(t^2+1)^3 + (t^2+1)^3 \dfrac{d}{dt}(\sin t)^2$

PRODUCT & CHAIN RULES

$= \boxed{(\sin^2 t)(3)(t^2+1)^2(2t) + (t^2+1)^3(2)(\sin t)(\cos t)}$

$= 2(\sin t)(t^2+1)^2\left[3t\sin t + (\cos t)(t^2+1)\right]$