

Math 131 - Homework 3

October 6, 2021

Name key
Score _____

The following problems are from the suggested homework. Show all work to receive full credit. Supply explanations when necessary. This assignment is due October 13.

1. (3 points) A potato is launched vertically upward with an initial velocity of 100 ft/s from a potato gun at the top of an 85-foot-tall building. The distance in feet that the potato travels from the ground after t seconds is given by $s(t) = -16t^2 + 100t + 85$.

(a) Determine the speed of the potato at 0.5 s.

$$v(t) = s'(t) = -32t + 100$$

$$v(0.5) = -16 + 100 = 84 \text{ FT/s}$$

(b) Determine when the potato reaches its maximum height.

$$v'(t) = 0 \Rightarrow -32t + 100 = 0$$

$$\Rightarrow t = \frac{100}{32}$$

$$t = 3.125 \text{ s}$$

(c) Determine the velocity of the potato as it hits the ground.

$$s(t) = 0 \Rightarrow -16t^2 + 100t + 85 = 0$$

$$t = \frac{-100 - \sqrt{10000 + 4(16)(85)}}{-32} \approx 7.008 \text{ s}$$

$$v(7.008) \approx -124 \text{ FT/s}$$

2. (2 points) Find the x -coordinates of the points at which the line tangent to the graph of $y = \left(x - \frac{6}{x}\right)^8$ is horizontal.

$$\frac{dy}{dx} = 8 \left(x - \frac{6}{x}\right)^7 \left(1 + \frac{6}{x^2}\right) = 0$$

$$x = \pm\sqrt{6}$$

$$x - \frac{6}{x} = 0 \quad \text{or} \quad x^2 - 6 = 0$$

Turn over.

3. (1 point) A mass hanging from a vertical spring is in simple harmonic motion as given by the following position function, where t is measured in seconds and s is in inches:

$$s(t) = -3 \cos\left(\pi t + \frac{\pi}{4}\right).$$

Find the velocity of the mass at $t = 1.5$ s.

$$v(t) = 3 \sin\left(\pi t + \frac{\pi}{4}\right) \cdot \pi$$

$$v(1.5) = 3\pi \sin\left(\frac{7\pi}{4}\right) = -\frac{3\pi\sqrt{2}}{2} \approx -6.66 \text{ in/s}$$

4. (3 points) The graph of the equation $2x^3 + 2y^3 - 9xy = 0$ is called a *folium of Descartes*. Find an equation of the line tangent to the graph at the point $(2, 1)$.

$$\frac{d}{dx}(2x^3 + 2y^3 - 9xy) = 0$$

$$6x^2 + 6y^2 \frac{dy}{dx} - 9x \frac{dy}{dx} - 9y = 0$$

$$\frac{dy}{dx} = \frac{9y - 6x^2}{6y^2 - 9x}$$

$$m = \frac{dy}{dx} \Big|_{(x,y)=(2,1)} = \frac{9 - 24}{6 - 18} = \frac{-15}{-12} = \frac{5}{4}$$

TAN LINE:

$$y - 1 = \frac{5}{4}(x - 2)$$

5. (1 point) Find $\frac{dy}{dx}$ if $y = \sin(\cos 7x)$.

$$\frac{dy}{dx} = \cos(\cos 7x) \cdot \frac{d}{dx}(\cos 7x)$$

$$= \cos(\cos 7x) \cdot (-\sin 7x) \cdot 7$$

CHAIN RULE TWICE.