

# Math 131 - Homework 4

October 27, 2021

Name key Score \_\_\_\_\_

The following problems are from the suggested homework. **Show all work to receive full credit.** Supply explanations when necessary. This assignment is due November 3.

1. (1 point) Let  $f(x) = x^3 + 2x + 3$ . Find  $(f^{-1})'(0)$ .

$$(f^{-1})'(0) = \frac{1}{f'(f^{-1}(0))} = \frac{1}{f'(-1)} = \boxed{\frac{1}{5}}$$

$$f^{-1}(0) = \omega \Leftrightarrow \omega^3 + 2\omega + 3 = 0 \Leftrightarrow \omega = -1, \quad f'(x) = 3x^2 + 2$$

2. (1 point) Use the following information to find  $(f^{-1})'(a)$ .

$$f(1) = -3, \quad f'(1) = 10, \quad a = -3$$

$$(f^{-1})'(-3) = \frac{1}{f'(f^{-1}(-3))} = \frac{1}{f'(1)} = \boxed{\frac{1}{10}}$$

$$f^{-1}(-3) = \omega \Leftrightarrow f(\omega) = -3 \Leftrightarrow \omega = 1$$

3. (1 point) Find  $\frac{dy}{dx}$  if  $y = \cos^{-1}(\sqrt{x})$ .

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{1}{2} x^{-1/2} = \boxed{\frac{-1}{2\sqrt{x}\sqrt{1-x}}}$$

4. (1 point) Find  $f'(x)$  if  $f(x) = \log_7(6x^4 + 3)^5$ .

$$f(x) = 5 \log_7(6x^4 + 3) = \frac{5}{\ln 7} \ln(6x^4 + 3)$$

$$f'(x) = \frac{5}{\ln 7} \frac{24x^3}{6x^4 + 3}$$

Turn over.

5. (1 point) Find  $f'(x)$  if  $f(x) = e^{x^3 \ln x}$ .

$$f'(x) = e^{x^3 \ln x} \cdot (x^3 + 3x^2 \ln x)$$

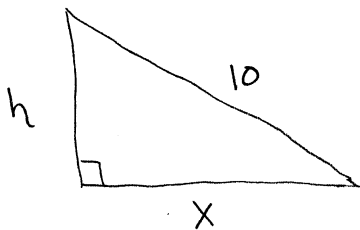
6. (2 points) Use logarithmic differentiation to find  $\frac{dy}{dx}$  when  $y = (\sin 2x)^{4x}$ .

$$\ln y = 4x \ln \sin 2x$$

$$\frac{1}{y} \frac{dy}{dx} = 4 \ln \sin 2x + 4x \left( \frac{2 \cos 2x}{\sin 2x} \right)$$

$$\frac{dy}{dx} = (\sin 2x)^{4x} \cdot (4 \ln \sin 2x + 8x \cot 2x)$$

7. (2 points) A 10-ft ladder is leaning against a wall. If the top of the ladder slides down the wall at a rate of 2 ft/sec, how fast is the bottom moving along the ground when the bottom of the ladder is 5 ft from the wall?



$$h^2 + x^2 = 100$$

$$2h \frac{dh}{dt} + 2x \frac{dx}{dt} = 0$$

$$\frac{dx}{dt} = -\frac{h}{x} \frac{dh}{dt}$$

$$\frac{dh}{dt} = -2$$

Find  $\frac{dx}{dt}$  when  $x = 5$

When  $x = 5 \dots$

$$h^2 = 75 \Rightarrow h = \sqrt{75}$$

AND  $\frac{dx}{dt} = -\frac{\sqrt{75}}{5} (-2)$

$$= 2\sqrt{3} \text{ FT/sec}$$

8. (1 point) Find the linearization of  $f(x) = \frac{1}{x}$  at  $x = 2$ .

$$f(a) = \frac{1}{2}$$

$$f'(x) = -\frac{1}{x^2}$$

$$f'(a) = -\frac{1}{4}$$

$$L(x) = \frac{1}{2} - \frac{1}{4}(x-2)$$