

Math 131 - Homework 5
December 1, 2021

Name key
Score _____

The following problems are from the suggested homework. **Show all work to receive full credit.** Supply explanations when necessary. This assignment is due December 8.

1. (4 points) Evaluate each limit.

(a) $\lim_{x \rightarrow \infty} x \sin\left(\frac{1}{x}\right)$ $\infty \cdot 0$ More work

$= \lim_{x \rightarrow \infty} \frac{\sin \frac{1}{x}}{\frac{1}{x}}$ % More work

L'Hôpital's Rule $= \lim_{x \rightarrow \infty} \frac{-\frac{1}{x^2} \cos\left(\frac{1}{x}\right)}{-\frac{1}{x^2}} = \cos 0 = \boxed{1}$

(b) $\lim_{x \rightarrow 0^+} x^{1/x}$

0^∞ Form is NOT INDETERMINATE!

Limit is $\boxed{0}$.

2. (2 points) Find the function $f(x)$ such that

$f'(x) = \frac{2}{x^2} - \frac{x^2}{2}$ and $f(1) = 0$.

$f(x) = \int \left(2x^{-2} - \frac{1}{2}x^2\right) dx = -2x^{-1} - \frac{1}{6}x^3 + C = -\frac{2}{x} - \frac{x^3}{6} + C$

$f(1) = 0 \Rightarrow -2 - \frac{1}{6} + C = 0 \Rightarrow C = \frac{13}{6}$

$f(x) = -\frac{2}{x} - \frac{x^3}{6} + \frac{13}{6}$

Turn over.

3. (2 points) Evaluate the indefinite integral: $\int (4\sqrt{x} + \sqrt[4]{x}) dx$.

$$\int 4x^{1/2} + x^{1/4} dx = \frac{4x^{3/2}}{3/2} + \frac{x^{5/4}}{5/4} + C$$

$$= \frac{8}{3} x^{3/2} + \frac{4}{5} x^{5/4} + C$$

4. (2 points) Let $f(x) = \frac{1}{x^2 + 1}$. Partition the interval $[-2, 2]$ into 4 subintervals of equal length. Using the left endpoints of the subintervals, compute the corresponding left Riemann sum for f .

$$\Delta x = \frac{2 - (-2)}{4} = 1 \Rightarrow \text{PARTITION: } -2 < -1 < 0 < 1 < 2$$

↑	↑	↑	↑
c_1	c_2	c_3	c_4

$$\text{RIEMANN SUM} = f(-2)(1) + f(-1)(1) + f(0)(1) + f(1)(1)$$

$$= \frac{1}{5} + \frac{1}{2} + 1 + \frac{1}{2}$$

$$= 2.2$$