Math 131 - Quiz 4

October 20, 2021

Name __key__

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due October 27.

1. (1 point) Let $g(x) = 3x - 7$. Compute $g^{-1}(8)$.

$$g^{-1}(8) = \omega$$
$$\updownarrow$$
$$g(\omega) = 8$$
$$\updownarrow$$
$$3\omega - 7 = 8 \iff \boxed{\omega = 5}$$

OR

$$g^{-1}(x) = \frac{x+7}{3}$$

$$g^{-1}(8) = \frac{15}{3} = 5$$

2. (3 points) For $x \geq 0$, let $f(x) = \sqrt{x^5 + 4}$. Find $(f^{-1})'(6)$.

$$\left(f^{-1}\right)'(6) = \frac{1}{f'(f^{-1}(6))}$$

$$f'(x) = \frac{1}{2}\left(x^5 + 4\right)^{-1/2}\left(5x^4\right)$$

$$f^{-1}(6) = \omega \iff \sqrt{\omega^5 + 4} = 6 \iff \omega = 2$$

$$f'(2) = \frac{1}{2}\left(36\right)^{-1/2}\left(5(16)\right)$$

$$= \frac{80}{12} = \frac{20}{3}$$

$$\left(f^{-1}\right)'(6) = \boxed{\frac{3}{20}}$$

3. (2 points) Let $f(x) = \ln(x^2 + 1)$. Find $f'(x)$ and use it to determine a point at which the graph's tangent line is horizontal.

$$f'(x) = \frac{2x}{x^2 + 1}$$

$$f'(x) = 0 \implies 2x = 0 \implies x = 0$$

$$\text{POINT}: \quad x = 0 \implies y = f(0) = \ln 1$$

$$= 0$$

$$\boxed{x = 0, \ y = 0}$$

Turn over.

4. (2 points) Find $g''(x)$ if $g(x) = e^{-5x^2}$.

$$g'(x) = e^{-5x^2}(-10x)$$

$$g''(x) = -10e^{-5x^2} + e^{-5x^2} \cdot (-10x)^2$$

$$g''(x) = -10e^{-5x^2} + 100x^2 e^{-5x^2}$$

5. (2 points) Use logarithmic differentiation to find $\dfrac{dy}{dx}$ when $y = \dfrac{(x+1)^2(x^3+1)}{4x^2(x-5)}$.

$$\ln y = 2\ln(x+1) + \ln(x^3+1) - \ln(4x^2) - \ln(x-5)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x+1} + \frac{3x^2}{x^3+1} - \frac{8x}{4x^2} - \frac{1}{x-5}$$

$$\frac{dy}{dx} = \frac{(x+1)^2(x^3+1)}{4x^3(x-5)}\left[\frac{2}{x+1} + \frac{3x^2}{x^3+1} - \frac{2}{x} - \frac{1}{x-5}\right]$$