

Math 131 - Quiz 5

November 3, 2021

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due November 10.

1. (2 points) Find the linearization of $g(x) = \sqrt{x^3 + 9}$ at the point where $x = 3$. Then use your linearization to approximate $g(2.9)$.

$$g(3) = \sqrt{36} = 6$$

$$g'(x) = \frac{1}{2} (x^3 + 9)^{-1/2} (3x^2)$$

$$g'(3) = \frac{27}{12} = \frac{9}{4}$$

$$L(x) = 6 + \frac{9}{4}(x-3)$$

$$g(2.9) \approx L(2.9)$$

$$= 6 + \frac{9}{4}(-0.1)$$

$$= 6 - \frac{9}{40}$$

$$= \boxed{5.775}$$

2. (3 points) Use calculus techniques to find the absolute extreme values of $f(x) = 3x^4 + 4x^3 - 72x^2$ on the interval $[-5, 5]$.

$$\begin{aligned} f'(x) &= 12x^3 + 12x^2 - 144x \\ &= 12x(x^2 + x - 12) \\ &= 12x(x+4)(x-3) = 0 \end{aligned}$$

$$\begin{aligned} \Downarrow \\ x=0, x=-4, x=3 \end{aligned}$$

CRIT. NUMBERS ARE $x=0, x=-4, x=3$

END PTS ARE $x=-5, x=5$

x	f(x)
0	0
-4	-640 ← ABS MIN
3	-297
-5	-425
5	575 ← ABS MAX

Turn over.

3. (2 points) Find the critical points of $f(x) = \sqrt[3]{x^2 - x}$.

$$f(x) = (x^2 - x)^{1/3}$$

$$f'(x) = \frac{1}{3} (x^2 - x)^{-2/3} (2x - 1)$$

$$f'(x) = 0 \Rightarrow 2x - 1 = 0 \Rightarrow \boxed{x = \frac{1}{2}}$$

$$f'(x) \text{ DNE} \Rightarrow x^2 - x = 0 \Rightarrow x(x - 1) = 0$$

$$\Rightarrow \begin{array}{l} x = 0 \\ x = 1 \end{array}$$

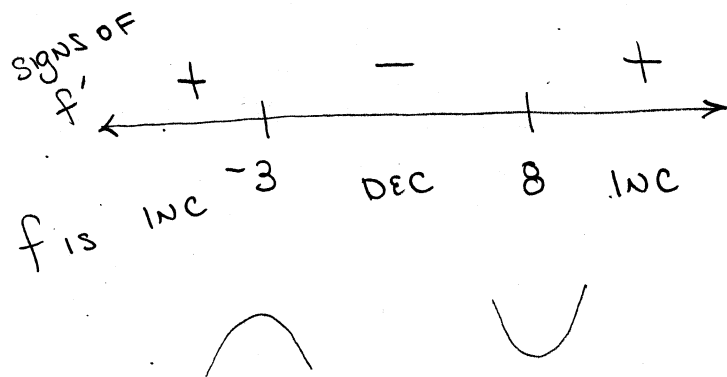
4. (3 points) Use calculus techniques to find open intervals on which

$$f(x) = \frac{1}{3}x^3 - \frac{5}{2}x^2 - 24x + 113 \text{ is increasing/decreasing.}$$

Also identify the relative extreme values.

$$f'(x) = x^2 - 5x - 24 = (x - 8)(x + 3)$$

$$f'(x) = 0 \Rightarrow x = 8, x = -3$$



f is increasing on

$$(-\infty, -3) \cup (8, \infty).$$

f is decreasing on $(-3, 8)$.

$$f(-3) = \frac{307}{2} \text{ is a REL. MAX.}$$

$$f(8) = -\frac{205}{3} \text{ is a REL. MIN.}$$