Math 131 - Quiz 6

November 17, 2021

Name Key

Score ______

Show all work to receive full credit. Supply explanations when necessary. This quiz is due December 1.

1. (2 points) Let $f(x) = x^3 - 6x^2 + 2x + 3$. Use calculus techniques to find open intervals on which the graph of f is concave up/down. Also identify all points of inflection (both coordinates).

$f'(x) = 3x^2 - 12x + 2$

$f''(x) = 6x - 12$

$f''(x) = 0 \Rightarrow x = 2$

SIGNS OF f'' — +

$x=2$

GRAPH IS CD CU

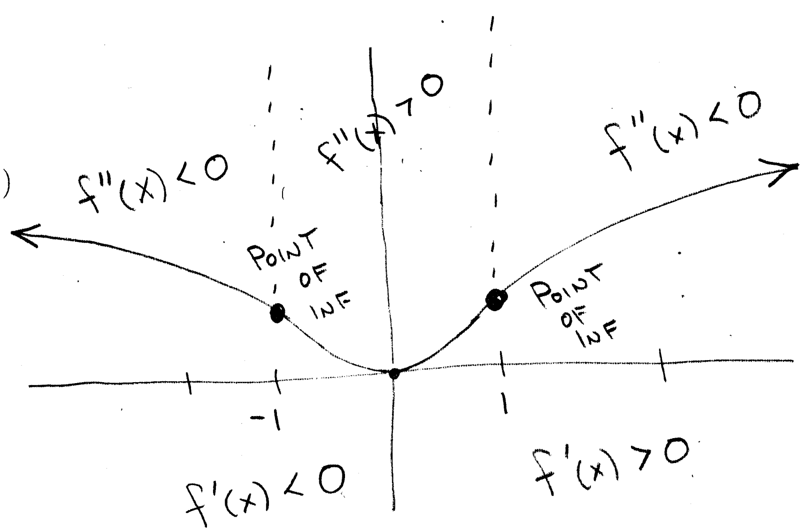
GRAPH IS CONCAVE DOWN ON $(-\infty, 2)$.

GRAPH IS CONCAVE UP ON $(2, \infty)$.

$(2, f(2)) = (2, -9)$ IS AN INFLECTION PT.

2. (2 points) Sketch the graph of a continuous function having all of the following properties.

- $f(0) = 0$, $f'(0) = 0$
- $f'(x) < 0$ on $(-\infty, 0)$
- $f'(x) > 0$ on $(0, \infty)$
- $f''(x) > 0$ on $(-1, 1)$
- $f''(x) < 0$ on $(-\infty, -1) \cup (1, \infty)$



Turn over.

3. (2 points) Find the limit, showing all work. Do not use L'Hôpital's rule.

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)\left(\frac{x^2+1}{x^2-1}\right)$$

∞/∞

$$= \underbrace{\lim_{x\to\infty}\left(1+\frac{1}{x}\right)}_{1} \cdot \underbrace{\lim_{x\to\infty}\left(\frac{x^2+1}{x^2-1}\right)\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}}}_{\lim_{x\to\infty}\frac{1+\frac{1}{x^2}}{1-\frac{1}{x^2}}} = 1\cdot 1 = \boxed{1.}$$

4. (2 points) Find the horizontal and vertical asymptotes of the graph of $h(x) = \dfrac{2-x^2}{x^2+x}$. Show work or explain your reasoning.

H.A....

∞/∞

$$\lim_{x\to\pm\infty}\frac{2-x^2}{x^2+x}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\pm\infty}\frac{\frac{2}{x^2}-1}{1+\frac{1}{x}} = -1$$

$y=-1$ IS THE H.A.

V.A....

$$h(x) = \frac{2-x^2}{x(x+1)}$$

$x=0$ gives $\frac{2}{0}$ FORM

$x=-1$ gives $\frac{1}{0}$ FORM

V.A.'S ARE $x=0$ & $x=-1$

5. (2 points) Use L'Hôpital's rule to find each limit.

(a) $\displaystyle\lim_{x\to 0}\frac{2x}{e^x-1}$ $\quad 0/0$

$$= \lim_{x\to 0}\frac{2}{e^x} = \frac{2}{1} = \boxed{2}$$

(b) $\displaystyle\lim_{x\to 1^+}\frac{\sin\pi x}{\sqrt{x-1}}$ $\quad 0/0$

$$= \lim_{x\to 1+}\frac{\pi\cos\pi x}{\frac{1}{2}(x-1)^{-1/2}} = \lim_{x\to 1+}\left(2\pi\sqrt{x-1}\cos\pi x\right)$$

$$= 2\pi(0)(-1) = \boxed{0}$$