Math 131 - Test 1
September 15, 2021

Name key

Score ______

Show all work to receive full credit. Supply explanations where necessary. When evaluating limits, you may need to use $+\infty$, $-\infty$, or DNE (does not exist). When classifying discontinuities, use the words *removable*, *nonremovable*, *infinite*, and/or *jump*.

1. (10 points) Think about our definition of *limit*. For each part below, does the table of values justify the given limit (yes or no)? Explain your reasoning.

(a) $\lim_{x\to 2} f(x) = 1$

x	2.1	2.01	2.001	2.0001	2.00001	2.000001
$f(x)$	1.098363	1.009998	1.001000	1.000100	1.000010	1.000001

$\boxed{\text{No.}}$ THE X-VALUES ARE ONLY TO THE RIGHT OF X=2.
THE TABLE ACTUALLY JUSTIFIES $\lim_{x\to 2^+} f(1) = 1$.

(b) $\lim_{x\to 0} g(x) = 7$

x	0.01	−0.01	0.001	−0.001	0.0001	−0.0001
$g(x)$	2.976341	3.001253	4.994369	5.000124	6.999998	7.000002

$\boxed{\text{No.}}$ THE X-VALUES ARE GETTING CLOSER AND CLOSER TO X=0, BUT THE Y-VALUES DO NOT INDICATE GETTING CLOSER & CLOSER TO 7. RATHER, THE Y-VALUES ARE ALL OF A SUDDEN CLOSE TO 7.

(c) $\lim_{x\to 3.5} h(x) = 8.5$

x	1.00	2.00	3.00	4.00	5.00	6.00
$h(x)$	8.342301	8.497634	8.499436	8.500293	8.522341	8.299863

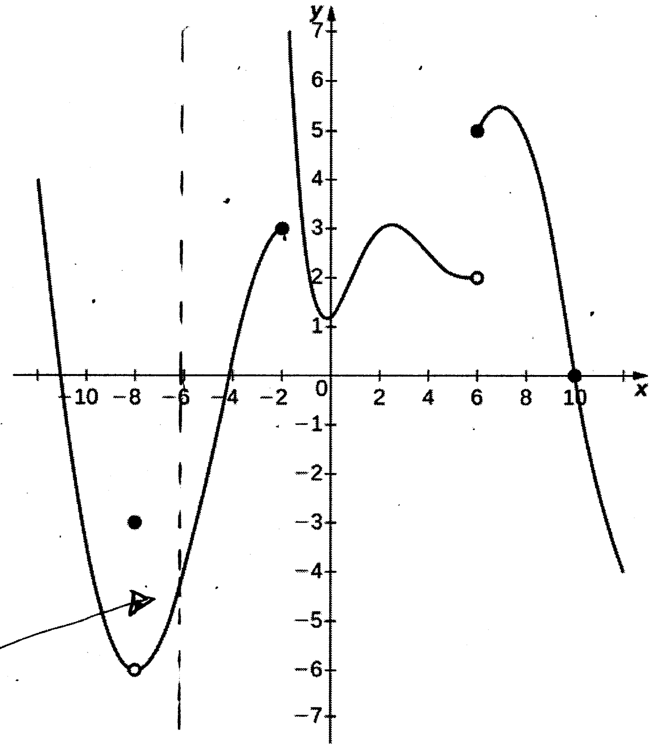
$\boxed{\text{No.}}$ THE X-VALUES ARE NOT GETTING CLOSER AND CLOSER TO X=3.5. IT'S IRRELEVANT WHAT THE Y'S ARE DOING.

(d) $\lim_{x\to 4^+} f(x) = \infty$

x	3.9	3.99	3.999	3.9999	3.99999	3.999999
$f(x)$	15.3	135.7	12,365.8	1,302,986.4	186,732,001.5	2,332,986,094.3

$\boxed{\text{No.}}$ THE TABLE ACTUALLY SEEMS TO JUSTIFY $\lim_{x\to 4^-} f(x) = \infty$.
THE X-VALUES ARE ON THE WRONG SIDE OF X=4.

2. (14 points) The graph of $y = f(x)$ is shown below. Use the graph to solve each part of this problem.



(a) What type of discontinuity does f have at $x = 6$? Explain your reasoning.

$\boxed{\text{Jump DISCONT.}}$ $\lim_{x \to 6^-} f(x) = 2 \neq 5 = \lim_{x \to 6^+} f(x)$

(b) Estimate $\lim_{x \to 10+} f(x)$.

$\boxed{0}$

(c) Estimate the value $f(-8)$.

$\boxed{-3}$

(d) Based on the graph, Steve believed that $\lim_{x \to -2^-} f(x) = \infty$. Do you agree or disagree? Explain your reasoning.

$\boxed{\text{DISAGREE.}}$ STEVE SHOULD GET 3. HE IS LOOKING FROM THE RIGHT OF $X = 3$, INSTEAD OF THE LEFT.

(e) Estimate $\lim_{x \to -6} f(x)$.

LOOKS LIKE $\boxed{-4.5 \text{ OR SO.}}$

(f) What type of discontinuity does f have at $x = -8$? Explain your reasoning.

$\boxed{\text{REMOVABLE.}}$ THE LIMIT EXISTS AT $X = -8$. LIMIT IS -6

(g) Estimate $\lim_{x \to 6^-} f(x)$.

LOOK LIKE $\boxed{2}$

3. (30 points) Determine each limit **analytically**, or explain why the limit does not exist. You may need to use $+\infty$, $-\infty$, or DNE.

(a) $\lim_{w\to 6} \frac{\sqrt{w+3}-3}{2w-12}$ % More work

$$\lim_{w\to 6} \frac{\sqrt{w+3}-3}{2(w-6)} \cdot \frac{\sqrt{w+3}+3}{\sqrt{w+3}+3} = \lim_{w\to 6} \frac{\cancel{w+3-9}}{2\cancel{(w-6)}(\sqrt{w+3}+3)} = \frac{1}{2\cdot 6} = \boxed{\frac{1}{12}}.$$

(b) $\lim_{x\to 2+} \frac{x^2+7x+10}{x^2+3x+2}$

$$= \frac{4+14+10}{4+6+2} = \frac{28}{12} = \boxed{\frac{7}{3}}$$

By DIRECT SUBSTITUTION

% More work

(c) $\lim_{h\to -5} \left(\frac{\frac{3}{h}+\frac{3}{5}}{h+5}\right)$

$$\lim_{h\to -5} \left(\frac{\frac{15+3h}{5h}}{h+5}\right) = \lim_{h\to -5} \frac{3\cancel{(h+5)}}{5h\cancel{(h+5)}} = \boxed{-\frac{3}{25}}$$

(d) $\lim_{y\to 0} \frac{y}{(y+6)^2-36}$ % More work

$$\lim_{y\to 0} \frac{y}{y^2+12y} = \lim_{y\to 0} \frac{1}{y+12} = \boxed{\frac{1}{12}}$$

(e) $\lim_{x\to 0} \frac{\tan 3x}{6x}$ % More work

$$\lim_{x\to 0} \frac{\sin 3x}{2\cdot 3x \cdot \cos 3x} = \lim_{x\to 0} \left(\frac{1}{2} \cdot \frac{\sin 3x}{3x} \cdot \frac{1}{\cos 3x}\right)$$

$$= \frac{1}{2}\cdot 1 \cdot 1 = \boxed{\frac{1}{2}}$$

4. (12 points) These limits DO NOT EXIST. **Choose any three (3) of them**, and clearly tell why the limit fails to exist. If necessary, provide evidence.

(a) $\lim_{x\to 7} \frac{9x}{(x-7)^4}$ $\frac{63}{0}$ FORM $\Rightarrow$ SOME KIND OF INF LIMITS.

#2 -- FUNCTION VALUES GROW WITHOUT BOUND.

(b) $\lim_{x\to 0} x^2 \ln x$

#4 -- $\ln x$ IS NOT DEFINED WHEN $x < 0$

(c) $\lim_{x\to 0} \frac{x^2+3x}{|x|}$ $\lim_{x\to 0^+} \frac{x^2+3x}{|x|} = \lim_{x\to 0^+} \frac{x^2+3x}{x} = 3$

#1 -- LIMIT FROM LEFT $\neq$ LIMIT FROM RIGHT $\lim_{x\to 0^-} \frac{x^2+3x}{|x|} = \lim_{x\to 0^-} \frac{x^2+3x}{-x} = -3$

(d) $\lim_{x\to \pi^+} \left(\frac{x-3}{\tan x}\right)$ $\frac{\pi-3}{0}$ FORM $\Rightarrow$ SOME KIND OF INF LIMIT

#2 ... FUNCTION VALUES GROW WITHOUT BOUND.

5. (9 points) Suppose that $\lim_{x\to 3} f(x) = 4$ and $\lim_{x\to 3} h(x)$ exists. Determine each limit.

(a) $\lim_{x\to 3} [x^2 f(x) + h(x)\sin \pi x]$ $\sin 3\pi = 0$

$$\lim_{x\to 3} x^2 \cdot \lim_{x\to 3} f(x) + \lim_{x\to 3} h(x) \cdot \lim_{x\to 3} \sin \pi x$$

$$= 9 \cdot 4 + \left(\lim_{x\to 3} h(x)\right) \cdot 0 = \boxed{36}$$

(b) $\lim_{x\to 3} [(x-3)f(x)h(x)]$

$$\lim_{x\to 3} (x-3) \cdot \lim_{x\to 3} f(x) \cdot \lim_{x\to 3} h(x) = 0$$

0

(c) $\lim_{x\to 3} h(x)$ if $\underbrace{\lim_{x\to 3} \frac{f(x)}{h(x)}}$ does not exist

$$\frac{\lim_{x\to 3} f(x)}{\lim_{x\to 3} h(x)} = \frac{4}{?}$$

$\leftarrow$ MUST BE $\boxed{\text{ZERO}}$ IF DNE.

6. (9 points) In each problem below, determine whether the limit is $+\infty$, $-\infty$, or DNE. Show work or explain your reasoning.

(a) $\lim_{x \to -6^+} \left(\frac{2x+4}{x+6} \right)$ $\quad -8/0 \Rightarrow$ SOME KIND OF INF. LIMIT

TO THE RIGHT OF $X = -6$...

NUMER. IS NEG. AND DENOM IS POS. $\Rightarrow$ LIMIT IS $\boxed{-\infty}$.

(b) $\lim_{x \to 8} \frac{x^2}{(x-8)^2}$ $\quad \frac{64}{0} \Rightarrow$ SOME KIND OF INF. LIMITS.

FRACTION IS A SQUARE --- ALWAYS POSITIVE.

LIMIT IS $\boxed{+\infty.}$

(c) $\lim_{x \to 7} \left(\frac{x}{x-7} \right)$ $\quad \frac{7}{0} \Rightarrow$ SOME KIND OF INF. LIMITS.

TO THE LEFT OF $X = 7$...

NUMER. IS POS. AND DENOM IS NEG $\Rightarrow \lim_{X \to 7^-} \frac{X}{X-7} = -\infty$

TO THE RIGHT OF $X = 7$...

NUMER. IS POS. AND DENOM. IS POS $\Rightarrow \lim_{X \to 7^+} \frac{X}{X-7} = +\infty$

$\therefore$ LIMIT DNE.

7. (6 points) Find and classify the discontinuities of $F(x) = \frac{x^2-4}{(x+3)(x-2)}$. Show work or explain your reasoning.

$$F(x) = \frac{(X-2)(X+2)}{(X+3)(X-2)} = \frac{X+2}{X+3}, \quad X \neq 2$$

F HAS A REMOVABLE DISCONT AT $X = 2$.
LIMIT EXISTS AT $X = 2$.
LIMIT IS $4/5$.

F HAS AN INFINITE DISCONT AT $X = -3$.
" $\frac{\text{NONZERO}}{\text{ZERO}}$ " FORM AT $X = -3$.

8. (3 points) Given that $-x^2 \leq x^2 \cos\frac{1}{x} \leq x^2$ for $x \neq 0$, compute $\lim_{x\to 0} x^2 \cos\frac{1}{x}$. Explain your reasoning.

SINCE $\lim_{x\to 0}(-x^2) = \lim_{x\to 0} x^2 = 0$,

$$\boxed{\lim_{x\to 0} x^2 \cos\frac{1}{x} = 0 \text{ BY THE SQUEEZE THM.}}$$

9. (2 points) Give an example of a rational function whose graph has a hole at $x = 1$ and a vertical asymptote at $x = -1$.

$$\boxed{f(x) = \frac{x-1}{(x-1)(x+1)} = \frac{x-1}{x^2-1}}$$

LIMIT EXISTS AT $x=1$, SO THE DISCONT. THERE IS REMOVABLE.

- $2/0$ FORM AT $x=-1$, SO DISCONT. THERE IS INFINITE.

10. (5 points) Determine whether each statement is true (T) or false (F).

(a) T If f is continuous at $x = c$, then f has a limit at $x = c$.

(b) F If f has a limit at $x = c$, then f is defined at $x = c$.

(c) F If f has a removable discontinuity at $x = 1$, then the limit at $x = 1$ does not exist.

(d) F If $f(5) = 3$, then $\lim_{x\to 5} f(x) = 3$.

(e) F The limit of any basic trigonometric function can always be found by direct substitution.