Math 131 - Test 3
November 10, 2021

Name key

Score ________

Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) Let $f(x) = x^5 + x^3 + x - 3$. Compute $(f^{-1})'(0)$.

$$(f^{-1})'(0) = \frac{1}{f'(f^{-1}(0))} = \frac{1}{f'(1)} = \boxed{\frac{1}{9}}$$

WORK IS THERE

$$f^{-1}(0) = \omega \Leftrightarrow \omega^5 + \omega^3 + \omega - 3 = 0$$

$$\Leftrightarrow \omega = 1$$

$$\therefore f^{-1}(0) = 1$$

$$f'(x) = 5x^4 + 3x^2 + 1$$

$$f'(1) = 5 + 3 + 1 = 9$$

2. (7 points) Find the slope of the line tangent to the graph of $y = x \sec^{-1}(x^2)$ at the point where $x = 2$.

$$\frac{dy}{dx} = \sec^{-1}(x^2) + x\left(\frac{2x}{x^2\sqrt{(x^2)^2 - 1}}\right)$$

$$= \sec^{-1}(x^2) + \frac{2}{\sqrt{x^4 - 1}} \Rightarrow \left.\frac{dy}{dx}\right|_{x=2} = \sec^{-1}(4) + \frac{2}{\sqrt{15}}$$

$$\approx 1.8345$$

3. (5 points) Find $\frac{dy}{dx}$ if $y = 2^{\cos x}$.

$$\frac{dy}{dx} = 2^{\cos x} \cdot \ln 2 \cdot \frac{d}{dx} \cos x$$

$$= \boxed{2^{\cos x} \cdot \ln 2 \cdot (-\sin x)}$$

4. (5 points) Let $h(x) = \log_7(4x+5)^2$. Find $h'(x)$.

$$h(x) = \frac{2}{\ln 7} \ln(4x+5)$$

$$\boxed{h'(x) = \frac{2}{\ln 7} \frac{4}{4x+5}}$$

5. (8 points) Let $f(x) = \dfrac{x^x}{(x+2)(4x-1)}$. Use logarithmic differentiation to find $f'(x)$.

$$y = \frac{x^x}{(x+2)(4x-1)}$$

$$\ln y = \underbrace{x \ln x}_{\text{PRODUCT RULE}} - \ln(x+2) - \ln(4x-1)$$

$$\frac{1}{y}\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \ln x - \frac{1}{x+2} - \frac{4}{4x-1}$$

$$\frac{dy}{dx} = \frac{x^x}{(x+2)(4x-1)} \cdot \left[1 + \ln x - \frac{1}{x+2} - \frac{4}{4x-1}\right]$$

6. (6 points) A particle is moving along the graph of $x^2y = 12$ in such a way that $\dfrac{dx}{dt} = 8$. Find $\dfrac{dy}{dt}$ when $x = 2$.

$$x^2y = 12 \Rightarrow y = 12x^{-2}$$

$$\frac{dy}{dt} = -24x^{-3}\frac{dx}{dt}$$

$$\left.\frac{dy}{dt}\right|_{x=2} = -24(2)^{-3}(8) = \boxed{-24}$$

7. (6 points) Suppose that the infected region of an injury is circular, and its radius is growing at a rate of 1.3 mm/hr. Find the rate of change of area of the infected region when the radius is 3 mm. (The area of a circle is given by πr^2.)

A = AREA OF REGION AT TIME t

r = RADIUS OF REGION AT TIME t

$\frac{dr}{dt} = 1.3$. FIND $\frac{dA}{dt}$ WHEN $r = 3$.

$A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\left.\frac{dA}{dt}\right|_{r=3} = 2\pi(3)(1.3)$

7.8π mm/hr

≈ 24.5 mm/hr

8. (8 points) Use the linearization of $g(x) = e^{x^2-4}$ at $x = 2$ to approximate $g(1.9)$.

$g(2) = e^0 = 1$

$g'(x) = 2x e^{x^2-4}$

$g'(2) = 4e^0 = 4$

$L(x) = 1 + 4(x-2)$

$g(1.9) \approx L(1.9) = 1 + 4(-0.1)$

$= 0.6$

9. (8 points) Let $f(x) = 5x^{2/3} + x^{5/3}$. Find $f'(x)$ and use it to determine the critical numbers of f.

$f'(x) = \frac{10}{3}x^{-1/3} + \frac{5}{3}x^{2/3} = \frac{5}{3}x^{-1/3}(2+x)$

$f'(x) = \frac{5(2+x)}{3\sqrt[3]{x}}$

$f'(x) = 0 \Rightarrow x = -2$

$f'(x)$ DNE $\Rightarrow x = 0$

BOTH ARE DOMAIN INTERIOR PTS.

$x = -2$, $x = 0$ ARE THE CRIT. PTS.

10. (6 points) Let $y = \tan^{-1}\sqrt{x}$. Compute the differential dy.

$$\frac{dy}{dx} = \frac{1}{1+(\sqrt{x})^2} \cdot \frac{1}{2}x^{-1/2}$$

$$\Rightarrow \boxed{dy = \frac{dx}{2\sqrt{x}\,(1+x)}}$$

11. (11 points) Use calculus techniques to find the absolute maximum and minimum values of $g(x) = 5 + 4x^3 - 3x^4$ on the interval $[-1, 2]$.

$g'(x) = 12x^2 - 12x^3 = 12x^2(1-x)$

$g'(x) = 0 \Rightarrow x = 0,\ x = 1$

$g'(x)$ DNE NOWHERE ON $[-1,2]$

CRIT #'s: $x = 0,\ x = 1$

END PTS: $x = -1,\ x = 2$

x	$g(x)$	
0	5	
1	6	← ABS MAX
-1	-2	
2	-11	← ABS MIN

12. (10 points) Determine whether each statement is true (T) or false (F).

(a) F If $f(5)$ is the absolute maximum value of f, then $x = 5$ must be a critical number.

$x = 5$ MUST BE A CRIT. NUMBER OR A DOMAIN BOUNDARY POINT

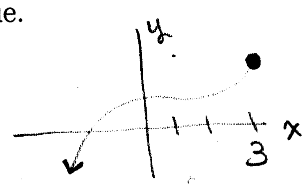
(b) F Every function must attain a maximum value.

$f(x) = x$ HAS NO MAX NOR MIN.

(c) T If the domain of f is the interval $[-1, 1]$, then it is impossible for $f(1)$ to be a relative maximum. RELATIVE EXTREME OCCUR ONLY AT DOMAIN INTERIOR POINTS.

(d) T If $f(2)$ is a relative minimum value of f, then $x = 2$ must be a critical number.

(e) T For a function defined at $x = 3$, it is possible for $f(3)$ to be an absolute maximum value but not a relative maximum value.



13. (14 points) Let $f(x) = x^4 - 8x^3 + 18x^2 - 11$.

(a) Use calculus techniques to find open intervals on which f is increasing/decreasing.

$$f'(x) = 4x^3 - 24x^2 + 36x = 4x\left(x^2 - 6x + 9\right) = 4x(x-3)^2$$

CRIT #'S ARE $X = 0$, $X = 3$

SIGNS OF f': $-$ | $+$ | $+$

f IS: DEC 0 INC 3 INC

f IS DECREASING ON $(-\infty, 0)$. f IS INCREASING ON $(0,3) \cup (3,\infty)$.

(b) Determine all relative extreme values of f.

$(0, f(0)) = \boxed{(0,-11) \text{ IS A RELATIVE MIN}}$

BECAUSE f DECREASES TO THE LEFT OF $X = 0$ AND INCREASES TO THE RIGHT.

(c) Based on your analysis in parts (a) and (b), as well as your skills from precalculus classes, what can you say about absolute extreme values of f?

THE GRAPH OF f HAS THE BASIC SHAPE

$f(0) = -11$ IS THE ABSOLUTE MIN.

THERE IS NO ABSOLUTE MAX.