

Math 131 - Test 3
November 10, 2021

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) Let $f(x) = x^5 + x^3 + x - 3$. Compute $(f^{-1})'(0)$.

2. (7 points) Find the slope of the line tangent to the graph of $y = x \sec^{-1}(x^2)$ at the point where $x = 2$.

3. (5 points) Find $\frac{dy}{dx}$ if $y = 2^{\cos x}$.

4. (5 points) Let $h(x) = \log_7(4x + 5)^2$. Find $h'(x)$.

5. (8 points) Let $f(x) = \frac{x^x}{(x+2)(4x-1)}$. Use logarithmic differentiation to find $f'(x)$.

6. (6 points) A particle is moving along the graph of $x^2y = 12$ in such a way that $\frac{dx}{dt} = 8$.
Find $\frac{dy}{dt}$ when $x = 2$.

7. (6 points) Suppose that the infected region of an injury is circular, and its radius is growing at a rate of 1.3 mm/hr. Find the rate of change of area of the infected region when the radius is 3 mm. (The area of a circle is given by πr^2 .)

8. (8 points) Use the linearization of $g(x) = e^{x^2-4}$ at $x = 2$ to approximate $g(1.9)$.

9. (8 points) Let $f(x) = 5x^{2/3} + x^{5/3}$. Find $f'(x)$ and use it to determine the critical numbers of f .

10. (6 points) Let $y = \tan^{-1} \sqrt{x}$. Compute the differential dy .
11. (11 points) Use calculus techniques to find the absolute maximum and minimum values of $g(x) = 5 + 4x^3 - 3x^4$ on the interval $[-1, 2]$.
12. (10 points) Determine whether each statement is true (T) or false (F).
- (a) _____ If $f(5)$ is the absolute maximum value of f , then $x = 5$ must be a critical number.
 - (b) _____ Every function must attain a maximum value.
 - (c) _____ If the domain of f is the interval $[-1, 1]$, then it is impossible for $f(1)$ to be a relative maximum.
 - (d) _____ If $f(2)$ is a relative minimum value of f , then $x = 2$ must be a critical number.
 - (e) _____ For a function defined at $x = 3$, it is possible for $f(3)$ to be an absolute maximum value but not a relative maximum value.

13. (14 points) Let $f(x) = x^4 - 8x^3 + 18x^2 - 11$.

(a) Use calculus techniques to find open intervals on which f is increasing/decreasing.

(b) Determine all relative extreme values of f .

(c) Based on your analysis in parts (a) and (b), as well as your skills from precalculus classes, what can you say about absolute extreme values of f ?