# Math 131 - Final Exam

Name _____

December 15, 2021

Score _____

**Show all work to receive full credit**. For each problem, place your final answer in the box provided. Each problem is worth 5 points–up to 2 points for the answer and up to 3 points for the supporting work or explanation.

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1. Determine the limit or briefly explain why it does not exist.

$$\lim_{x \to 2} \sqrt{4 - x^2}$$

2. Determine the limit. Show analytically (not with a graph or table) how you got your answer.

$$\lim_{x \to 0} \frac{1 + \cos x^3}{x(x + 8)}$$

3. Determine the limit. Use algebraic techniques (not a graph, table, or L'Hôpital's rule) to show how you got your answer.

$$\lim_{x \to 2} \left[ \frac{(x+2)^2 - 3x - 10}{x^2 - 2x} \right]$$

4. Yes or No: Is $f$ continuous at $x = -2$? Use the definition of continuity to support your answer.

$$f(x) = \begin{cases} 2x + \cos \pi x, & x < -2 \\ x - 1, & x \geq -2 \end{cases}$$

5. Find $\dfrac{d^2y}{dx^2}$ if $y = 5x^3 \sin x$.

6. Let $f(x) = x^3$. Write $f'(x)$ in the box, then use the limit definition of derivative to obtain your answer.

7. Find $f'(0)$ if $f(x) = (e^x + 2\cos x)^4$.

8. Find an equation of the line tangent to the graph of $x^2 + 2xy + y^4 = 5x - 1$ at the point $(2, 1)$.

9. Let $g(x) = \sin^{-1}(2x) + \tan^{-1}(x^2) + \cos^{-1}(2x)$. Determine $g'(x)$.

10. Some values of $f(x)$ and $f'(x)$ near $x = 1$ are given in the table below.

| $x$ | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 |
|---|---|---|---|---|---|
| $f(x)$ | 6.08 | 6.90 | 8.00 | 9.41 | 11.14 |
| $f'(x)$ | 2.74 | 3.82 | 5.00 | 6.26 | 7.60 |

Determine the linearization of $f$ at $x = 1$, and use it to approximate $f(0.9)$.

11. Find the slope of the line tangent to the graph of $y = \log_5(3x + 2)^4$ at the point where $x = 1$. Write your answer in decimal form, rounded to the nearest thousandth.

12. Find the critical number(s) of $f(x) = \dfrac{x^2 + 4}{x}$.

13. Evaluate the limit: $\lim_{x \to \infty} x^2 e^{-x}$

14. Use calculus techniques to find the absolute extreme values of $f(x) = 3x^4 - 8x^3 - 48x^2$ on $[-3, 1]$.

15. The second derivative of $f$ is given by $f''(x) = (3x-1)(x+2)e^{15x}$. Find open intervals on which the graph of $f$ is concave up.

16. Find the function $f$ for which $f'(x) = e^x + \sqrt{x} + \dfrac{2}{x}$ and $f(1) = 1$.

17. Let $f(x) = 2x$. Use 6 subintervals of equal length and subinterval midpoints (for the $c_k$'s) to compute a Riemann sum for $f$ on $[1, 4]$.

18. Evaluate the definite integral: $\displaystyle\int_0^2 (2x^3 - 7x^2 + 4x - 1)\,dx$

19. Find the area of the region between the graph of $y = 1 + \sin x + \cos x$ and the $x$-axis over the interval $[0, \pi]$.

20. Use a $u$-substitution to evaluate the indefinite integral: $\displaystyle\int x(4x^2 - 1)^5\,dx$