Math 131 - Quiz 10

November 30, 2022

Name key

Score

Show all work to receive full credit. Supply explanations when necessary. This quiz is due December 5.

1. (2 points) Determine the function f that satisfies $f'(x) = x^3 - 2x^2 + 7x$ and $f(1) = 4$.

$$f(x) = \frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{7}{2}x^2 + C$$

$$f(1) = 4 \Rightarrow \frac{1}{4} - \frac{2}{3} + \frac{7}{2} + C = 4$$

$$C = 4 - \frac{1}{4} + \frac{2}{3} - \frac{7}{2} = \frac{11}{12}$$

$$f(x) = \frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{7}{2}x^2 + \frac{11}{12}$$

2. (3 points) Let $f(x) = \frac{1}{x}$. Use 6 subintervals of equal length and subinterval left endpoints to compute the corresponding Riemann sum for f over the interval $[1, 4]$.

$$\Delta x = \frac{4-1}{6} = \frac{1}{2}$$

$c_1 = 1$

$c_2 = \frac{3}{2}$

$c_3 = \frac{4}{2}$

$c_4 = \frac{5}{2}$

$c_5 = \frac{6}{2}$

$c_6 = \frac{7}{2}$

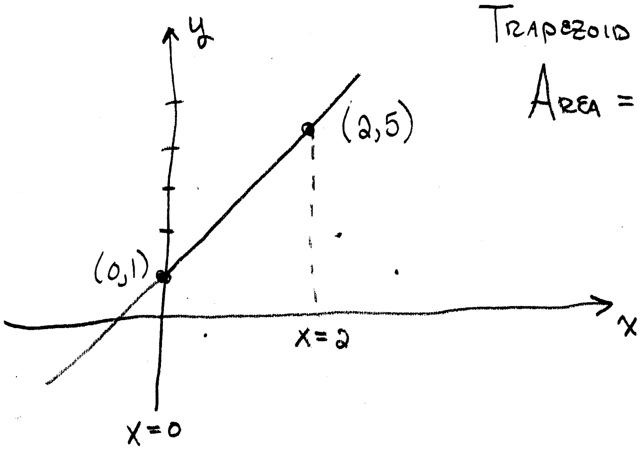
$$\text{Riemann sum} = \sum_{k=1}^{6} f(c_k)\,\Delta x$$

$$= \frac{1}{2}\left(\frac{2}{2} + \frac{2}{3} + \frac{2}{4} + \frac{2}{5} + \frac{2}{6} + \frac{2}{7}\right)$$

$$= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} = \frac{223}{140} \approx 1.592857$$

Turn over.

3. (2 points) Use the area concept to evaluate $\int_0^2 (2x+1)\,dx$. Show your work.



TRAPEZOID

$$\text{AREA} = \frac{5+1}{2}\cdot(2-0)$$

$$= \boxed{6} = \int_0^2 (2x+1)\,dx$$

4. (3 points) Use the fundamental theorem of calculus to evaluate each definite integral.

(a) $\int_0^{\pi/2} (x+\sin x)\,dx$

$$= \frac{1}{2}x^2 - \cos x \Big|_0^{\pi/2} = \left(\frac{\pi^2}{8} - 0\right) - (0-1)$$

$$= \frac{\pi^2}{8} + 1$$

(b) $\int_1^2 \frac{1+x}{x}\,dx = \int_1^2 \left(\frac{1}{x}+1\right)dx = \ln x + x \Big|_1^2$

$$= (\ln(2) + 2) - (\ln 1 + 1)$$

$$= \ln 2 + 1$$