Math 131 - Quiz 11

December 7, 2022

Name key

Score

Show all work to receive full credit. Supply explanations when necessary. This quiz is due December 12.

1. (4.5 points) Consider the function $F(x) = \int_x^2 (t + t^2 + t^3)\,dt$.

   (a) Evaluate $F(1)$.

$$F(x) = \int_x^2 (t+t^2+t^3)\,dt = \frac{1}{2}t^2 + \frac{1}{3}t^3 + \frac{1}{4}t^4 \Big|_x^2$$

$$= \left(\frac{4}{2} + \frac{8}{3} + \frac{16}{4}\right) - \left(\frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4\right)$$

$$= \frac{26}{3} - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4$$

$$F(1) = \frac{26}{3} - \frac{1}{2} - \frac{1}{3} - \frac{1}{4} = \boxed{\frac{91}{12} \approx 7.58\overline{3}}$$

   (b) Find $F'(x)$ by evaluating the integral and then differentiating.

$$F(x) = \frac{26}{3} - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4 \quad \text{(FROM ABOVE)}$$

$$\boxed{F'(x) = -x - x^2 - x^3}$$

   (c) Find $F'(x)$ by using the 2nd Fundamental Theorem of Calculus.

$$\frac{d}{dx}\int_x^2 (t+t^2+t^3)\,dt = -\frac{d}{dx}\int_2^x (t+t^2+t^3)\,dt$$

$$= \boxed{-(x+x^2+x^3)}$$

Turn over.

2. (1.5 points) Evaluate the indefinite integral $\int \left(\frac{2}{x} + \sec^2 x - \frac{1}{x^2+4}\right) dx$.

$$= 2\ln|x| + \tan x - \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$

3. (4 points) Use a substitution to evaluate each definite integral.

(a) $\int_0^1 6x^2(x^3+12)^{118}\,dx \quad = 2\int_{12}^{13} u^{118}\,du$

$u = x^3 + 12$

$du = 3x^2\,dx$

$2\,du = 6x^2\,dx$

$$= \frac{2}{119} u^{119} \Big|_{12}^{13}$$

$$= \frac{2}{119}\left[13^{119} - 12^{119}\right]$$

$\approx 6.1 \times 10^{130}$

(b) $\int_1^2 3x\,e^{x^2}\,dx \quad = \frac{3}{2}\int_1^4 e^u\,du = \frac{3}{2}e^u \Big|_1^4$

$u = x^2$

$du = 2x\,dx$

$\frac{3}{2}\,du = 3x\,dx$

$$= \frac{3}{2}\left[e^4 - e\right]$$

≈ 77.8198