Math 131 - Quiz 2 (IC)

August 31, 2022

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Determine each limit analytically.

(a) $\lim_{x\to 2}\left(\dfrac{x^3+3x^2-10x}{x^2+x-6}\right)$ $\overset{0/0}{=}$ $\lim_{x\to 2}\dfrac{x(x-2)(x+5)}{(x-2)(x+3)}$

$= \lim_{x\to 2}\dfrac{x(x+5)}{x+3} = \boxed{\dfrac{14}{5}}$

(b) $\lim_{w\to 0}\dfrac{\tan w}{5w}$ $\overset{0/0}{=}$ $\dfrac{1}{5}\lim_{w\to 0}\dfrac{\tan w}{w} = \dfrac{1}{5}\lim_{w\to 0}\dfrac{\sin w}{w\cos w}$

$= \dfrac{1}{5}\cdot\lim_{w\to 0}\dfrac{\sin w}{w}\cdot\lim_{w\to 0}\dfrac{1}{\cos w}$

$= \dfrac{1}{5}(1)(1) = \boxed{\dfrac{1}{5}}$

2. (2 points) Explain why the substitution technique cannot be used to determine the limit. Then evaluate the limit analytically.

$$\lim_{x\to 1}\frac{(x-3)^2-4}{x-1}$$

Direction substitution yields $0/0$. Cannot use direction subs if it results in a zero denominator!

$\lim_{x\to 1}\dfrac{x^2-6x+9-4}{x-1} = \lim_{x\to 1}\dfrac{x^2-6x+5}{x-1} = \lim_{x\to 1}\dfrac{(x-1)(x-5)}{x-1}$

$= \boxed{-4}$

Math 131 - Quiz 2 (TH)

August 31, 2022

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due September 7.

1. (3 points) Evaluate each limit analytically.

(a) $\lim_{x\to -3^+} \dfrac{2-\sqrt{x^2-5}}{x+3}$ %

$= \lim_{x\to -3^+} \dfrac{2-\sqrt{x^2-5}}{x+3} \cdot \dfrac{2+\sqrt{x^2-5}}{2+\sqrt{x^2-5}}$

$= \lim_{x\to -3^+} \dfrac{4-(x^2-5)}{(x+3)(2+\sqrt{x^2-5})} = \lim_{x\to -3^+} \dfrac{9-x^2}{(x+3)(2+\sqrt{x^2-5})}$

$= \lim_{x\to -3^+} \dfrac{(3-x)\cancel{(3+x)}}{\cancel{(x+3)}(2+\sqrt{x^2-5})} = \dfrac{6}{2+\sqrt{4}} = \dfrac{6}{4} = \boxed{\dfrac{3}{2}}$

(b) $\lim_{x\to 0^-} \dfrac{x^5+7x}{|x|}$ %

For $x<0$, $|x| = -x$

$\lim_{x\to 0^-} \dfrac{x^5+7x}{-x} = \lim_{x\to 0^-} -(x^4+7) = \boxed{-7}$

2. (2 points) Evaluate the limit analytically. Use $+\infty$, $-\infty$, or DNE if appropriate.

$\lim_{x\to 2} \dfrac{3x^2-5x-2}{(x-2)^2}$ 0% more work $= \lim_{x\to 2} \dfrac{\cancel{(x-2)}(3x+1)}{\cancel{(x-2)}(x-2)}$ 7% some kind of ∞

Just to the left of $x=2$,

$\dfrac{3x+1}{x-2} = \dfrac{\text{pos}}{\text{neg}} = \text{neg}$

$\Downarrow$

$\lim_{x\to 2^-} \dfrac{3x+1}{x-2} = -\infty$

Just to the right of $x=2$,

$\dfrac{3x+1}{x-2} = \dfrac{\text{pos}}{\text{pos}} = \text{pos}$

$\Downarrow$

$\lim_{x\to 2^+} \dfrac{3x+1}{x-2} = +\infty$

The conclusion from the one-sided limits is that the two-sided limit DNE.