Math 131 - Quiz 3

September 7, 2022

Name Key

Score ________

Show all work to receive full credit. Supply explanations when necessary. This quiz is due September 12.

1. (2 points) Find the number k so that f is continuous at $x = 0$.

$$f(x) = \begin{cases} \dfrac{\tan 3x}{5x}, & x < 0 \\ x^2 + x + k, & x \geq 0 \end{cases}$$

WE MUST HAVE

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0).$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{\tan 3x}{5x} = \lim_{x \to 0^-} \frac{3}{5} \frac{\sin 3x}{3x} \cdot \frac{1}{\cos 3x} = \frac{3}{5} \cdot 1 \cdot 1 = \frac{3}{5}$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x^2 + x + k = k = f(0)$$

$$\boxed{k = \frac{3}{5}}$$

2. (3 points) Find and classify the discontinuities of $r(x) = \dfrac{x^2 + 6x - 7}{x^2 - 1}$.

SEE ATTACHED SHEET.

Turn over.

3. (2 points) Find an interval of length one that contains a solution of the equation $x^5 - 3x^3 + 7x - 13 = 0$. Use the Intermediate Value Theorem to explain your answer.

SEE ATTACHED SHEET.

4. (3 points) Argue that the following function is continuous everywhere.

$$g(x) = \begin{cases} 5x + \cos(\pi x) + e^{x-1}, & x \le 1 \\ 3x^2 - 7x + 9, & x > 1 \end{cases}$$

EACH "PIECE" IS CONTINUOUS EVERYWHERE.

WE ONLY NEED TO WORRY ABOUT WHERE ONE PIECE LEAVES OFF AND THE OTHER PICKS UP.

THE ONLY POSSIBLE DISCONT. IS AT $X = 1$.

LET'S CHECK IT...

$$\left.\begin{aligned} f(1) = \lim_{x \to 1^-} f(x) &= 5(1) + \cos(\pi) + e^0 = 5 \\ \lim_{x \to 1^+} f(x) &= 3(1)^2 - 7(1) + 9 = 5 \end{aligned}\right\}$$

SAME. NO DISCONT AT $X = 1$.

QUIZ 3 – QUESTION [redacted] 3

LET $f(x) = x^5 - 3x^3 + 7x - 13$.

f IS CONTINUOUS FOR ALL VALUES OF X.

NOTICE THAT $f(1) = (1)^5 - 3(1)^3 + 7(1) - 13$

$= 1 - 3 + 7 - 13 = -8.$

ALSO NOTICE THAT $f(2) = (2)^5 - 3(2)^3 + 7(2) - 13$

$= 32 - 24 + 14 - 13 = 9$

SINCE $f(1) = -8$ AND $f(2) = 9$,

f MUST TAKE ON ANY y-VALUE BETWEEN

-8 AND 9 ON THE INTERVAL WHERE

$1 \leq x \leq 2$.

IN PARTICULAR, $f(x)$ MUST BE ZERO

SOMEWHERE BETWEEN $X = 1$ AND $X = 2$.

Quiz 3 – Question [redacted] 2

$$f(x) = \frac{x^2 + 6x - 7}{x^2 - 1} = \frac{(x+7)(x-1)}{(x+1)(x-1)}$$

There are two discontinuities arising from a zero denominator: $x = -1$, $x = 1$

$x = -1$:

$$\lim_{x \to -1} \frac{(x+7)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} = \underbrace{\lim_{x \to -1} \frac{(x+7)}{(x+1)}}$$

This gives a $\frac{6}{0}$ form.

This is enough to know that $\boxed{x = -1 \text{ is a nonremovable, infinite discont.}}$

$x = 1$

$$\lim_{x \to 1} \frac{(x+7)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} = \frac{8}{2} = 4 \qquad \text{Limit exists!}$$

$$\boxed{x = 1 \text{ is a removable discont.}}$$