Math 131 - Quiz 4 (IC)

September 21, 2022

Name: key

Score: ______

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Let $f(x) = 3x - x^2$. Use the limit definition of derivative to determine $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[3(x+h) - (x+h)^2] - [3x - x^2]}{h}$$

$$= \lim_{h \to 0} \frac{3x + 3h - x^2 - 2xh - h^2 - 3x + x^2}{h}$$

$$= \lim_{h \to 0} \frac{3h - 2xh - h^2}{h} = \lim_{h \to 0} (3 - 2x - h)$$

$$= 3 - 2x$$

$$\boxed{f'(x) = 3 - 2x}$$

Math 131 - Quiz 4 (TH)

September 21, 2022

Name key

Score

Show all work to receive full credit. Supply explanations when necessary. Unless otherwise indicated, use differentiation rules rather than the limit definition of derivative. This quiz is due September 26.

1. (3 points) Determine each derivative.

(a) $\frac{d}{dx}(5x^4 - 17x + 7)$.

$$= \boxed{20x^3 - 17}$$

(b) $\frac{d}{dx}\left(\frac{1}{x^3} + 5\cos x\right) = \frac{d}{dx}\left(x^{-3} + 5\cos x\right)$

$$= \boxed{-3x^{-4} - 5\sin x = \frac{-3}{x^4} - 5\sin x}$$

(c) $\frac{d}{dx}\left(\frac{8}{\sqrt[3]{x^2}} + 3\sin x\right) = \frac{d}{dx}\left(8x^{-2/3} + 3\sin x\right)$

$$= \boxed{-\frac{16}{3}x^{-5/3} + 3\cos x = \frac{-16}{3\sqrt[3]{x^5}} + 3\cos x}$$

2. (2 points) Find an equation of the line tangent to the graph of $y = x^{1/2} + x^2 + x^{-1/2}$ at the point where $x = 4$.

$$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} + 2x - \frac{1}{2}x^{-3/2}$$

$$m = \left.\frac{dy}{dx}\right|_{x=4} = \frac{1}{2}(4)^{-1/2} + 8 - \frac{1}{2}(4)^{-3/2}$$

$$= \frac{1}{4} + 8 - \frac{1}{16} = \frac{131}{16}$$

Point: $x = 4$, $y = (4)^{1/2} + (4)^2 + (4)^{-1/2} = \frac{37}{2}$

Tangent line:

$$y - \frac{37}{2} = \frac{131}{16}(x - 4)$$

or

$$y = \frac{131}{16}x - \frac{57}{4}$$

Turn over.

3. (2 points) Use the limit definition of the derivative to determine $g'(x)$ when $g(x) = \frac{1}{x}$.

$$g'(x) = \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h\to 0} \left[\frac{x}{(x+h)x} - \frac{x+h}{(x+h)x}\right]\frac{1}{h}$$

$$= \lim_{h\to 0} \frac{-\cancel{h}}{(x+h)(x)(\cancel{h})} = \lim_{h\to 0} \frac{-1}{(x+h)(x)}$$

$$= -\frac{1}{x^2}$$